Maximizing the Fair Allocation of Opportunistic Spectrum for CR Ad hoc Networks

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***Abstract*—Cognitive Radios (CRs) address spectrum scarcity and under-utilization of the spectrum. Allocating spectrum to CR nodes in a network is neither easy nor straight-forward since spectrum availability is space and time-varying. The Link layer always sets up communication between nodes in a CR ad hoc network by allocating the available spectrum suitably. In a time-slotted model, allocation of channels amongst CR nodes is a $\mathcal{NP}$ complete problem. Moreover, since the availability of spectrum is time varying, all the nodes in the network may not be allocated sufficient resources. Thus there is always a possibility that a node may starve compared to its counterparts. In this article, we address the issue of fairness in a CR ad hoc network and propose novel distributed heuristics to allocate spectrum fairly. In literature, max-min, max-sum and max-proportional fairness based schemes are proposed to provide fairness in these CR ad hoc networks. We compare our heuristic results with these schemes, and discuss why our method is significantly better.**

***Index Terms*—Cognitive Radio Ad hoc networks, MAC, scheduling, distributed heuristics, fairness**

I. INTRODUCTION

The current regulatory practice of exclusively licensing frequency bands has resulted in over-crowding of some bands while many licensed bands are under-utilized, thus creating an artificial scarcity of spectrum. Spectrum utilization is only 15% to 85% as reported by FCC. DARPA reports an average of 6% licensed spectrum usage [1]. *Cognitive Radios* (CRs) have been proposed to solve this artificial scarcity. A CR infers from its environment choices such as transmission parameters, spectrum band, etc., to communicate. They can detect unused spectrum bands (spectrum holes), and access them *opportunistically*. With *Opportunistic Spectrum Access* (OSA), DARPA [1] envisages a ten fold increase in spectrum utilization.

A *Cognitive Radio Ad hoc Network* (CRAN), formed by CR nodes, and its components of a CRAN architecture are shown in [2]. CRAN nodes (*Secondary Users* or SUs) can opportunistically access spectrum bands or channels (both licensed and free bands) for communication. Hereafter, we use the terms channel and spectrum interchangeably throughout this paper. Since available channels and SUs in a CRAN may vary over time and space and often no central entity exists for coordination, it is a challenge to *fairly* allocate spectrum in CRANs. Moreover, SUs should abide by policies and etiquettes that are in force. Important challenges in spectrum sharing [3], [4] are: (a) Spectrum coordination and access; (b) Variation in Spectrum availability; (c) Distributed allocation; and (d) Fairness.

As a first step, we proposed a distributed, time-slotted allocation method in [5], where both sender and receiver are made aware of the schedule and the transmitter-receiver handshake is achieved. While trying to obtain an optimal allocation in [5] we neglected limitations on the frame size. In reality, while trying to simply maximize utilization, some nodes may starve due to limited resources (channels and slots). Thus this brings in the issue of fairness, which is an important criterion. If fairness is ensured then some form of QoS could also be supported.

II. EXISTING SOLUTIONS

Graph coloring is usually proposed for channel allocation in cellular networks, where each base station is allocated with a non-interfering channel with respect to its neighboring base stations. It is known that both edge and node coloring of graphs are $\mathcal{NP}$ complete problems [6] in general. Several edge coloring heuristics have been proposed for link scheduling in multi-hop wireless networks [7] [8]. These solutions if applied to CRANs will result in degraded performance, since all of them assume static channel availability. A better approach of spectrum allocation with fairness was proposed in Zheng's work [4] on fairness. In [4], a weighted, color-sensitive graph $G(U, E_C, L_B)$ is used to represent the topology of the CRANs. where vertices U, a set of colored edges E_C, and link weights L_B. An optimization framework is proposed with the graph coloring and utility functions to optimize bandwidth and fairness. The spectrum allocation for CRs is developed based on vertex coloring; each vertex is assigned a set of channels (colors) based on the 'Progressive Minimum Neighbor First' heuristic. The utility functions govern the fairness achieved. Three functions are defined: (a) *Max-Sum Bandwidth* - to maximize the total spectrum utilization in the system; (b) *Max-Min-Bandwidth* - to maximize the bottleneck user's spectrum utilization; and (c) *Max-Proportional-Fair* - to obtain a fair

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allocation. Both cooperative and non-cooperative approaches are discussed. Centralized and distributed solutions are proposed that implement the utility functions. It is shown that cooperative approaches perform better than non-cooperative methods, and the distributed approaches perform almost as good as the centralized one. While this work deals with many aspects of fair allocation it has two unrealistic assumptions: (i) it looks only at the topology-optimized fair allocation functions; and (ii) users can utilize any number of channels at any time. It does not describe the process of transmitter-receiver handshake either. Thus in this article we present a framework that addresses all the three issues with a distributed and fair spectrum sharing mechanism. We compare our heuristics with the collaborative schemes proposed in this paper i.e., collaborative max-min (CMIN), collaborative max-sum (CSUM) and collaborative max-proportional-fair (CFAIR).

III. PROBLEM DESCRIPTION

A. System model and assumptions

We consider a multi-hop CR network formed by N nodes with node-ids $\in \{1, 2, ...N\}$. These nodes are equipped with a half-duplex software-defined transceiver. We consider only 1-hop communication. We assume that the spectrum is divided into M orthogonal channels that are symmetric. Each channel has a unique number in $\{1, 2, ...M\}$. Further, we assume that a local common control channel (CCC) is available to exchange the control messages amongst the nodes. The nodes are time synchronized through the CCC. *We assume that all nodes have data to transmit to all its neighbors to ensure that we consider the worst case scenario of the problem*. However the algorithm is agnostic to this.

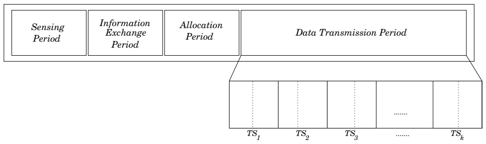

Fig. 1. MAC super frame

Consider the MAC super frame as shown in Fig. 1. The super frame is divided into sensing, information exchange, allocation and data transmission periods. In the sensing period, the channels that are free of PUs are found. We assume perfect sensing by nodes (either with or without cooperative sensing) for the sake of simplicity. The nodes then send the list of channels available to a central node or to their neighbors during the information exchange period. In the spectrum allocation period, algorithms are executed to determine the schedule and the schedule is distributed. The data transmission period is split into $\mathcal{K}$ equal length time-slots. In a time-slot TS_k both the participating nodes transmit and receive data i.e., the time-slot consists of two sub-slots which are shared by the two corresponding nodes to transmit. This assumption reduces the complexity in the design, since each link has to be assigned with only one time-slot now. Channel switching times can be apportioned within the sub-slots. For time synchronization, a scheme such as [9] can be used. We assume that PUs change states only at the beginning of the frame. However, these assumptions do not change the problem of fair sharing of spectrum on hand. It can be clearly seen that this framework addresses all the problems of [4] as mentioned in Section II.

To determine $\mathcal{K}$, it is easy to see that for the node with maximum degree (or maximum neighbors) to communicate with all neighbors in one frame, $\mathcal{K}$ should be greater than or equal to $MaxDegree(\Delta)$, where $MaxDegree$ is the maximum degree of the graph. However, with varying topology, if $\mathcal{K}$ varies every frame, then there are problems to confront (specially when we seek a distributed approach to allocating fairly): (a) The information about the topology (maximum degree and number of channels) should be known by all nodes. (b) Since $\mathcal{K}$ is an upper bound some slots remain unused. Exchanging topology information each frame results in higher overheads. Hence we would like to fix $\mathcal{K}$ to Δ, and maximize utilization with fair allocation. By fairness, we mean *all transmitter-receiver pairs should approximately be allocated with the same percentage of access to the medium over sufficient time*.

B. Problem formulation

We provide a generalized formulation of an allocation problem. Let the CRAN be represented by an undirected graph $G = (V, E)$, with no self-loops, where V is the set of vertices or nodes, and E is the set of links between vertices in V. A link exists between two vertices x and y if both x and y are in communicating radii of each other and at least one common channel is available to them. The spectrum allocation problem is then to find an optimal valid assignment of $(timeslot, channel)$ pair for every link.

$$A_{(x,y),j,k} = \begin{cases} 1 & \text{if } (x,y) \text{ uses channel } j \text{ in timeslot } k \leq \mathcal{K} \\ 0 & \text{otherwise} \end{cases}$$

such that,

i) x and y are in communication radii of each other,
ii) both x and y are not involved in any other communication in TS_k,
iii) assignment of channel j does not cause interference to neighboring nodes.

Let $\mathbf{A}$ represent an adjacency matrix of the allocation such that,

$$\mathbf{A}_{(x,y)} = \begin{cases} (k, j) & \text{if } A_{(x,y),j,k} = 1 \\ 0 & \text{otherwise} \end{cases}$$

Here we consider long-term fairness. The allocation is said to be *fair* over H frames if all the edges have received equal number of opportunities to access the spectrum, as $H \to \infty$. Further the allocation in each frame has to be *optimal*[1]. Since optimality and fairness can not be seen in isolation, we address them together. This problem of spectrum allocation is $\mathcal{NP}$ complete in general, and hence we propose heuristics.

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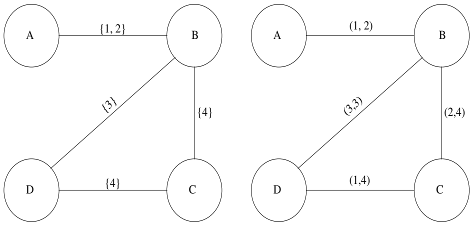
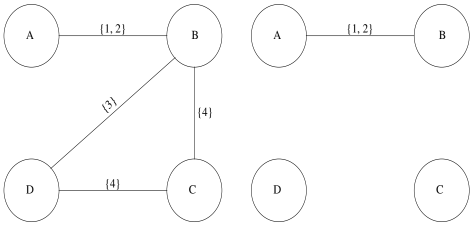
[1] If the length of allocation $\mathbf{A}$, $Len(\mathbf{A})$ equal to the number of slots in the allocation, then it is optimal if the $Len(\mathbf{A})$ is the least over all possible allocations.

C. Problem Illustration

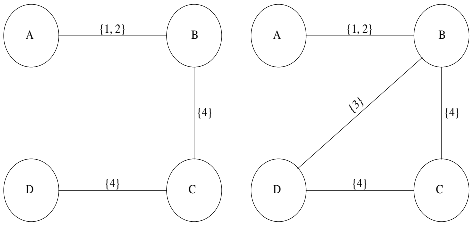
Consider the CRAN in Fig. 2(a). The channels available for each link are shown. A valid schedule for this graph is shown in Fig. 2(b), which is of length 3. Let the number of slots, $\mathcal{K}$, in data transmission period per each frame be 2. The nodes periodically sense with period T. Let the network at time $t = pT$, where $p = \{0, 1, 2, 3\}$, be as shown in Fig. 2. Using any of the techniques described in [5], the schedule length, L_s, will be 3. By computing a schedule using an algorithm in [5] for every $\lceil L_s/\mathcal{K} \rceil$ frames, the schedule is spread across multiple frames. This cannot ensure fairness because of the dynamic topologies created due to the PU activity, which is also illustrated in Fig. 2.



(a) A CRAN. (b) A valid assignment



(c) At $t = 0$ (d) At $t = T$



(e) At $t = 2T$ (f) At $t = 3T$

Fig. 2. Snapshots of the graph at frame times $t = 0, T, 2T, 3T$; The available channels between nodes are indicated within brackets

The first schedule is computed with Fig. 2(d) at $t = 0$ with schedule length 3. Fig. 3(a) shows the allocation resulting from an algorithm of [5] split into two frames. The empty slots can be used for free-slot communication. Over time as topology changes, possible allocations without a fairness component in the algorithm are shown in Fig. 3. It can be seen that link (B, D) is never given a chance while (A, B) link is allocated the most. This *unfairness* should be reduced, which is the main task of this article.

IV. PROPOSED HEURISTICS

A. Clique Based Heuristic

In [5] we have proposed the Clique Based Heuristic (CBH) which is not only distributed but also achieves close to optimal allocation. In CBH, two hop topology information is exchanged between the nodes initially. Based on the node-id and degree of each node independently determines whether assigning a $(timeslot, channel)$ pair to its edges is its responsibility. The assignment of $(timeslot, channel)$ pair is done in a greedy fashion. Once the assignment is done, each node exchanges the schedule in its one hop neighbourhood. Since the allocation is done considering local information distributedly, conflicts are bound to arise. They are resolved using a simple conflict resolution procedure.

Frame 1		Frame 2	
Slot 1	Slot 2	Slot 1	Slot 2
(A, B) (C, D)	(B, C)	(B, D)	

(a) Allocation split into two frames

Frame 1	
Slot 1	Slot 2
(A, B) (C, D)	(B, C)

(b) Frame/schedule computed at $t = 0$

Frame 3	
Slot 1	Slot 2
(A, B)	(A, B)

(c) Allocation for schedule computed at $t = T$

Frame 3	
Slot 1	Slot 2
(A, B) (C, D)	(B, C)

(d) Possible allocation for schedule computed at $t = 2T$

Frame 4	
Slot 1	Slot 2
(A, B) (C, D)	(B, C)

(e) Possible allocation for schedule computed at $t = 3T$

Fig. 3. Inefficiency of using schedule across frames

B. Fair Clique Based Heuristic

Here, we propose Fair-Clique based heuristic (F-CBH) based on CBH. To quantify fairness, we define *Fairness Index* (FI) for an edge as,

$$f_{(x,y)} = \frac{\Sigma A_{(x,y),j,k}}{n_{(x,y)}} \quad (x, y) \in E \; and \; \forall j, k, \qquad (1)$$

where $n_{(x,y)}$ is the total number of times edge (x, y) existed[2].

The fairness index indicates the fraction of times an edge was given access to the channel. If an edge uses an available free-slot for communication, this also has to be taken into account to calculate the fairness index. The problem now is to ensure that all edges get the same share.

Without getting into details we outline the steps involved in fair allocation (refer to [5] for pseudo-code. It must be noted that those procedures are modified, with modifications presented here. We regret not presenting them here due to lack of space). The procedures employed here are simple. The algorithm of F-CBH is shown in Alg. 1. First the procedure to form a clique set is executed, in which each node decides if it is part of clique based on degrees of itself and its neighbors. Each node opts itself out of the clique if its degree

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[2] An edge exists between x and y if there exists at least one available channel between them to communicate.

is the least among its one hop neighbors. The nodes in the clique have the responsibility of assigning its edges with a $(timeslot, channel)$ pair. For the edges between two clique nodes, the node with lower node-id assigns slot and channel.

The Slot and Channel Assignment procedure implements a simple greedy algorithm – the set of responsible edges for a node are sorted in ascending order of their fairness indices, and first $\mathcal{K}$ edges of the sorted list are assigned with a slot and a channel.

Conflicts are detected when the schedules are distributed. Here the two-hop neighborhood schedule is learnt by every node. The procedure for conflict resolution is that the node with least Node-ID proposes to find a new non-interfering channel if possible, or proposes to use a new slot. In F-CBH, if a new slot has to be proposed then the new slot is less than $\mathcal{K}$. The other end-vertex receives the proposal, and also computes another valid allocation for this edge. There can be zero, one or two proposals. In case there are two, the best one i.e., one which has lower slot, is chosen. If there is one proposal, it is chosen to resolve the conflict. In case there are none, then out of the interfering edges, the edge with lower fairness index retains the allocation and the other one loses. In case both have same fairness index, then the edge having the least node-id retains the allocation and the other edge loses. In this way, priority is given to the starving edges to ensure fairness.

Algorithm 1 A Simple Fair Clique Based Heuristic (F-CBH)

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Input: Topology and available channels for all edges
Output: Allocation Matrix
Form Clique Set
Assign Slots and Channels first to the edges which have lesser FI using (1)
Distribute schedule over two-hop neighborhood
Resolve conflicts
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We illustrate how fairness is achieved across edges using the previous example. We consider the same graph with four nodes as in Fig. 2. We show the evolution of allocation for 5 frames pictorially. The changes to the graph due to the PU activities shown in Fig. 2 at time instances $t = 0, T, 2T, 3T$ is also carried here. The number of slots $\mathcal{K}$ is 2. The snapshots over time is shown in Fig.4. We allocate freely available channels to the edges which have no conflicts and require extra channels after we try to allocate them to the edges which have lesser FI.

Complexity: The worst-case time complexity for Form Clique Set procedure is $\mathcal{O}(N)$; Slot and Channel Assignment procedure is $\mathcal{O}(MN\mathcal{K})$, and Conflict resolution procedure is $\mathcal{O}(MN^2\mathcal{K})$.

V. RESULTS AND DISCUSSIONS

A. Simulation environment and Experiments setup

We performed simulations to test the algorithms for various metrics. The simulator creates a 2D area with four PUs. The dimensions of the area, maximum number of channels, and number of SUs are input by the user. The SUs are uniformly distributed over the area. Each PU has a call arrival rate of 36 calls/hour with mean call duration of 80s. Due to the exponential on-off distribution of channel occupancy, dynamic topologies of the SU nodes are created based on the availability of channels for a SU pair. Further, due to the effect of shadowing and fading the SUs may detect the channels as free or occupied based on channel characteristics. Thus number of channels varies between neighbors. We should note that the fairness index is also the average utilization

Node	Value
(B,A)	0
(B,C)	0
(B,D)	0
(D,C)	0

Edge-fairness

Slot 1	Slot 2
(A,B)	(B,C)
(C,D)	

Allocation

(a) Allocation for Frame 1

Node	Value
(B,A)	1
(B,C)	1
(B,D)	0
(D,C)	1

Edge-fairness

Slot 1	Slot 2
(A,B)	(B,A)

Allocation

(b) Allocation for Frame 2

Node	Value
(B,A)	3/2
(B,C)	1
(B,D)	0
(D,C)	1

Edge-fairness

Slot 1	Slot 2
(C,B)	(B,A)
	(C,D)

Allocation

(c) Allocation for Frame 3

Node	Value
(B,A)	4/3
(B,C)	1
(B,D)	0
(D,C)	1

Edge-fairness

Slot 1	Slot 2
(D,B)	(B,C)

Allocation

(d) Allocation for Frame 4

Node	Value
(B,A)	1
(B,C)	1
(B,D)	1/2
(D,C)	2/3

Edge-fairness

Slot 1	Slot 2
(D,B)	(B,C)

Allocation

(e) Allocation for Frame 5

Fig. 4. Snapshots of the Allocation for frames at times $t = 1, 2, 3, 4, 5$

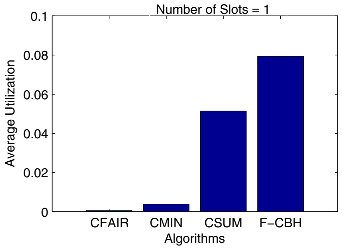

Fig. 5. Average Utilization obtained by different algorithms

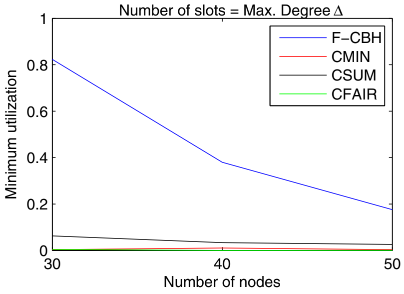

Fig. 6. Minimum utilization v/s number of nodes

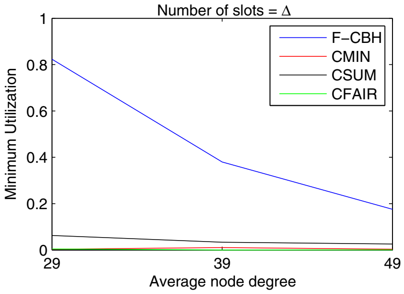

Fig. 7. Minimum utilization v/s average node degree

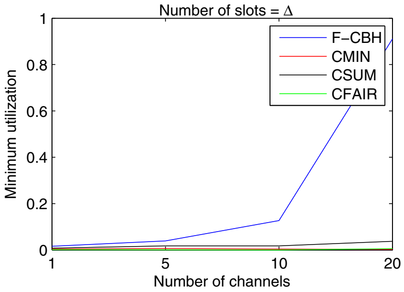

Fig. 8. Minimum utilization v/s number of channels

of an edge. It is also the case in [4], where they define fairness as reward (or utilization). We compare our results with the algorithms in [4]. Particularly, we compare Collaborative Max-Min Fairness (CMIN), Collaborative Max-Sum Fairness (CSUM) and Collaborative Max-Proportional Fair (CFAIR) algorithms.

B. *Results and Discussions*

Fig. 7 shows the average fairness index per edge for randomly generate graphs of 50 nodes. We compare the average utilization of our algorithm with the above mentioned algorithms in [4]. The main reason for F-CBH performing better than the others is that we allow transmissions on parallel edges since this a valid allocation in a TDMA system and the use of sub-slots. We enhance the allocations in [4] by packing tightly in each frame using sub-slots. Note that we have used only one sub-slot in this comparison, since we use edge-coloring mechanism and [4] uses vertex coloring. Hence to obtain a comparison scale, we use one sub-slot.

In Fig. 8 we compare the least utilization obtained in the graph. The least utilization indicates the least allocation i.e., we can see how *fair* the allocation is. Since with one sub-slot it is possible that at least one of the links of the graph can be unallocated, we increase the number of sub-slots to Δ of the graph. It is fair to compare with [4] since we allow a vertex to have more than one color. We find the our algorithm outperforms those in [4]. As number of nodes increases the density increases, hence the number of conflicts increase. So we can see that utilization reduces with increasing number of nodes. In Fig. 9 we compare the least utilization with respect to average node degree. We see higher performance since with increasing node degree the density of links increases, hence the number of conflicts increase. So we can see that utilization reduces with increasing number of nodes. In Fig. 10 we compare the least utilization with respect to number of channels. As number of channels increases, more opportunity is available for allocation which is depicted in Fig. 10.

VI. CONCLUSIONS

The main aim of this article is to introduce and propose solutions to the problems involved in allocation of spectrum opportunistically but fairly amongst the nodes in a CRAN. SUs indeed require some level of fair allocation since they are competing for the channels left by PUs. We touched upon important aspects that need to be looked into carefully while allocating channels. In this article we formulated the problem of fair allocation of channels in a CRAN in a time-slotted framework. The framework solves the problem of transmitter-receiver handshake. We propose F-CBH, a distributed allocation algorithm which is modified version of our previous algorithms in [5]. We compare our solutions with CMIN, CSUM and CFAIR algorithms of [4]. We find that our algorithm performs significantly better than the ones in the literature. F-CBH, as we have seen, turns out to be the better heuristic. Two main reasons for this: (a) a time-slotted framework allows allocation on parallel edges that increase the utilization of the system, and thus in turn helps increase fairness among nodes. This parallel edge allocation is being done in F-CBH; (b) we try to maximize the utilization by allocating a link whenever possible - as we call it "freeslot-allocation" in [5]. We can conclude our framework yields better results.

ACKNOWLEDGMENT

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