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On the lifetime of node-to-node communication in wireless ad hoc networks ☆

Javad Vazifehdan *, R. Venkatesha Prasad, Ignas Niemegeers

Delft University of Technology, Mekelweg 4, 2628 CD, Delft, The Netherlands

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ABSTRACT

Lifetime of node-to-node communication in a wireless ad hoc network is defined as the duration that two nodes can communicate with each other. Failure of the two nodes or failure of the last available route between them ends their communication. In this paper, we analyze the maximum lifetime of node-to-node communication in static ad hoc networks when alternative routes that keep the two nodes connected to each other are node-disjoint. We target ad hoc networks with random topology modeled as a random geometric graph. The analysis is provided for (1) networks that support automatic repeat request (ARQ) at the medium access control level and (2) networks that do not support ARQ. On the basis of this analysis, we propose numerical algorithms to predict at each moment of network operation, the maximum duration that two nodes can still communicate with each other. Then, we derive a closed-form expression for the expected value of maximum node-to-node communication lifetime in the network. As a byproduct of our analysis, we also derive upper and lower bounds on the lifetime of node-disjoint routes in static ad hoc networks. We verify the accuracy of our analysis using extensive simulation studies.

1. Introduction

Node redundancy in wireless ad hoc networks provides a degree of fault-tolerance and increases reliability of data communication. That is, after the first route between two nodes fails, there might be alternate routes through other intermediate nodes to keep the connectivity. A source node can transfer data to a destination node until they both are alive and there is at least one route between them (see Fig. 1). Here, the basic question is how long any two arbitrary nodes in a wireless ad hoc network with a random topology can communicate with each other without interruption due to lack of routes between them before their own batteries run out. This we call here as node-to-node communication lifetime.

Analysis of node-to-node communication lifetime in ad hoc networks allows us to identify how and to what extent various factors such as node distribution, transmission range, network deployment area, packet transmission rate, and energy consumption characteristics of nodes affect the lifetime. This can help us to define configurable parameters such that the communication lifetime of nodes is maximized. Although many studies have addressed problems related to this problem in wireless ad hoc networks, a complete analysis considering all factors such as network connectivity and energy consumption characteristics of nodes is missing. The problem of connectivity of ad hoc networks has been studied extensively in [1,22,23,21,8,12,17,25,24,9]. This problem deals with estimating the probability that an ad hoc network is k-connected as well as finding the minimum node density and the minimum transmission range required to keep the network k-connected. However, to determine the node-to-node communication lifetime in ad hoc networks we have to consider the energy consumption rate of nodes as well. Assuming

☆ Some preliminary results have been published in [30].

* Corresponding author. Tel.: +31 152786446; fax: +31 152781774.
E-mail addresses: j.vazifehdan@tudelft.nl, javad.vazifehdan@gmail.com (J. Vazifehdan).

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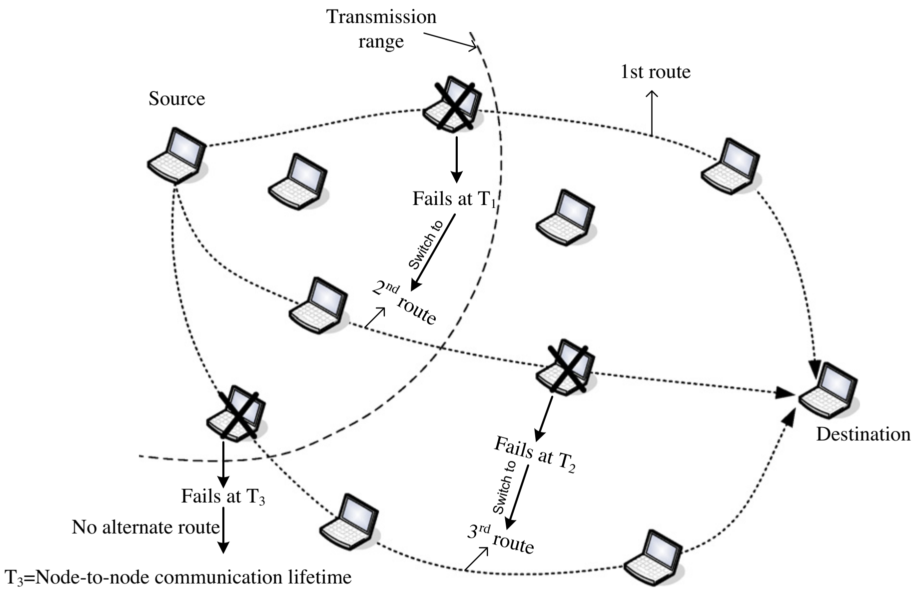

Fig. 1. An illustration of node-to-node communication lifetime in ad hoc networks.

the network is k-connected, the energy consumption rate of nodes determines when the first route, the second, and ultimately the kth node-disjoint route between two nodes fail due to the battery exhaustion of their intermediate nodes.

Analysis of the network lifetime is another problem related to the problem of node-to-node communication lifetime in wireless ad hoc networks. This relation depends on the definition of the network lifetime. Multitude definitions of network lifetime have been studied. Some, for example, define the network lifetime in terms of time until the first node fails due to energy drain [15] or time until the first cluster head is drained of energy [27]. Although failure of a node in a global scale may contribute to disconnection of a pair of nodes from each other, we should notice that due to redundancy in wireless ad hoc networks, failure of a node or a cluster head may not result in lack of end-to-end communication between two specific nodes. Other perspectives found in the literature specify the network lifetime as the time until a fraction of nodes are survived in the network [6] or the time until an area of interest is covered by a subset of nodes [5,2,4,32]. Knowing these values of the network lifetime can provide us insights about the duration of node-to-node connectivity in the network. However, they do not provide precise information, because connectivity of nodes is not considered explicitly. Studies which are more relevant to our work in this paper define the network lifetime in terms of the network connectivity. Examples are the duration that the network remains k-connected [16,26], or the duration that the size of the largest connected component of the network remains above a threshold [3]. Nevertheless, we should notice that these connectivity-based definitions of the network lifetime have been studied in terms of communication to a base station in wireless sensor networks, which is different from the ability of an arbitrary pair of nodes to communicate in a general purpose ad hoc network. This requires analyzing the lifetime of alternative routes connecting two nodes to each other. A similar problem has been tackled by Zhang et al. [33] and Tseng et al. [28]. They proposed algorithms to predict the duration that a route stays intact when the nodes are mobile. However, we know that battery drain of nodes may also cause route failure. Furthermore, communication between two nodes might be ended not only due to route failure but also due to depletion of their own batteries. Our analysis of the node-to-node communication lifetime in this paper covers all these missing aspects. Furthermore, what we analysis as node-to-node communication lifetime could be considered as a variant of the network lifetime. That is, it considers the lifetime of mutual connectivity of nodes as an indicator of the network lifetime. Node-to-node communication lifetime can also reflect the time until the network is partitioned, because two nodes get disconnected due to network partitioning.

Node-to-node communication lifetime in ad hoc networks also has some relation with the resilience of these networks. Najjar and Gaudiot [19] defined network resilience as the maximum number of node failures (say for example, due to battery exhaustion) which can be tolerated while the network remains connected with a certain probability. There are also some other definitions provided for the network resilience in the literature. Colboum [7] defined the network resilience as the expected number of node pairs that can communicate with each other when some nodes fail in the network. Ganesan et al. [11] defined it as the likelihood that an alternative route is available between a source and a destination when the shortest path between them fails. Finally, Dimitar et al. [10] defined the network resilience as the maximum number of nodes that can be removed from the network such that the probability that a connection is available between a pair of nodes remains above a threshold. Considering the concept of resilience in ad hoc networks, we may say that node-to-node communication lifetime is the duration that a connection between two nodes remains resilient. To find this, we need to know when the two nodes disconnect from each other due to failure of intermediate nodes between them. Among various studies, only Xing and Wang [31] analyzed the expected duration that a node is connected to the rest of the network. Nevertheless, they modeled node failure as a Markov chain to analyze the connectivity of a mobile node to its neighboring nodes in a steady state, without considering the energy consumption characteristics of nodes and failure of routes due to failure of their intermediate nodes.

Our contribution in this paper is to address the node-to-node communication lifetime in static ad hoc networks by modeling and analyzing the maximum duration that two nodes can stay connected to each other. The analysis is provided for networks in which the MAC (medium access control) layer supports automatic repeat request (ARQ) to recover lost packets, and networks in which ARQ is not supported. ARQ affects the energy consumption rate of nodes due to retransmission of lost packets. This, in turn, affects the node-to-node communication lifetime in the network. Our main contributions in this paper are as follows:

- We propose numerical algorithms to calculate the communication lifetime of two specific nodes in networks with and without ARQ. The presented algorithms can predict at any moment the maximum duration that two arbitrary nodes can still communicate with each other. To this end, we model energy consumption rate and remaining battery energy of nodes at any instance of the network operation.
- We derive a closed-form expression for the expected value of maximum node-to-node communication lifetime. The expression that we derive can predict the expected maximum duration that two arbitrary nodes in a static ad hoc network with a random topology can communicate with each other.
- We derive upper and lower bounds for the lifetime of alternate node-disjoint routes between nodes.
- Using extensive simulation studies, we show the accuracy of our analysis. We show that the numerical algorithms and the closed-form expression can accurately estimate communication lifetime of nodes. We also verify the accuracy of upper and lower bounds on the lifetime of node-disjoint routes.

The highlight of this work is that our treatment here is comprehensive in approach considering all the minute details.

The rest of the paper is structured as follows: In Section 2, we present preliminaries. In Section 3, we provide a more specific definition for node-to-node communication lifetime in ad hoc networks. The energy consumption rate of nodes is analyzed in Section 4 for networks without ARQ support, and in Section 5 for networks with ARQ support. The numerical algorithms to calculate the communication lifetime of nodes are presented in these two sections as well. We derive upper and lower bounds on the lifetime of node-disjoint routes in Section 6. In Section 7, we derive a closed-form expression for the expected communication lifetime of nodes. Simulation results are presented in Section 8. We conclude in Section 9.

2. Preliminaries

2.1. Network model

Consider topology of a wireless ad hoc network represented by a graph $G(\mathbb{V},\mathbb{E})$, where $\mathbb{V}$ and $\mathbb{E}$ are the set of vertices (nodes) and edges (links), respectively. We assume $N = |\mathbb{V}|$ nodes are uniformly distributed in the network area. Thus, the network topology could be random. If the Euclidean distance between two nodes u and v in the network is less than d_{max} (the transmission range), we assume that there is a link between them (i.e., $(u, v) \in \mathbb{E}$). Note that the network topology may not necessarily be a connected graph. It may consist of a number of disconnected subgraphs. Here, we consider a general case without making any assumption about connectivity of the network topology. Connectivity of ad hoc networks depends on density and transmission range of nodes (this will be discussed further in Section 7).

We assume nodes are static. There could be several applications of ad hoc networking in which nodes are static. For example, in a home network, appliances could form an ad hoc network to exchange context [20]. Other examples of static ad hoc networks are wireless sensor networks and wireless mesh networks.

We define $\mathbb{B}(t) = \{B_u(t)\}_{u=1}^{N}$ as the set of battery energy of nodes at time t, where $B_u(t)$ denotes the battery energy of node $u \in \mathbb{V}$ at time t in Joule.

Definition 1 (*Node failure*). If the residual battery energy of a node falls bellow a threshold B_{th}, the node is considered to be failed. Without loss of generality, we assume $B_{th} = 0$.

As we assumed nodes are failed due to battery exhaustion, other types of failure of nodes such as malicious attacks are excluded from our analysis. Communication failure between nodes is only due to battery exhaustion of nodes. Furthermore, since we study static ad hoc networks, communication failure due to mobility of nodes is not considered in this work as well.

2.2. Node-disjoint routes

Definition 2 (*Node-disjoint routes*). Two routes between a pair of source–destination nodes are node-disjoint, if they don't have any intermediate node in common.

We assume multiple routes used to keep nodes connected to each other are node-disjoint [18]. In node-disjoint routes, failure of a node results in failure of only one of the available routes between the source and the destination. This reduces the frequency of route discovery and saves energy, because such routes could be discovered once before the communication between two nodes starts. In non-disjoint routes with common intermediate nodes, failure of a single node may result in failure of several routes. Thus, the routing protocol may require to trigger

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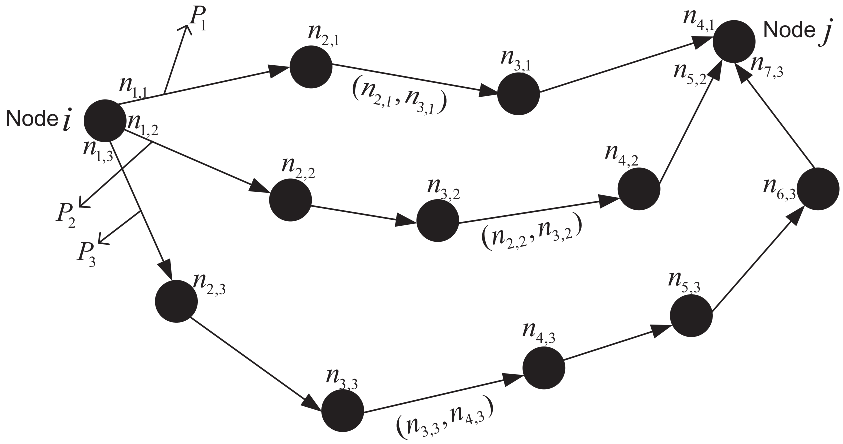

Fig. 2. Illustration of the node-disjoint routes between a source node, *i*, and a destination node, *j*. In this figure, there are three routes between the source and the destination, (i.e., $K_{i,j} = 3$). Depending on the in-use route, different notations are considered for the source and the destination node.

a new route discovery whenever a node in non-disjoint routes fails.[1] This, indeed, generates a high overhead and increases energy consumption of nodes for route discovery. For this reason, many multi-path routing protocols try to discover node-disjoint routes [11,29,18]. How such routing protocols discover node-disjoint routes is out of the scope of this paper. Interested readers are referred to [18] for more information about multi-path routing in wireless ad hoc networks. In this paper, we only analyze the duration that two nodes can communicate with each other using node-disjoint routes.

We denote $P_k = \langle n_{1,k}, n_{2,k}, \ldots, n_{h_k,k}, n_{h_k+1,k} \rangle$ as the *k*th node-disjoint route between the source and the destination node, in which $n_{l,k}$ is the *l*th node in P_k and h_k is the number of hops of P_k. In each route, $n_{1,k}$ is the source node, $n_{h_k+1,k}$ is the destination node, and $n_{2,k}, \ldots, n_{h_k,k}$ are intermediate (relay) nodes which forward packets hop by hop from the source to the destination. The number of node-disjoint routes between the source node *i* and the destination node *j* is denoted by $K_{i,j}$. Note that we use different notations for the same source and destination nodes in different node-disjoint routes. That is, $n_{1,1}, n_{1,2}, \ldots, n_{1,K_{i,j}}$ all refer to the source node *i*. Similarly, $n_{h_1+1,1}, n_{h_2+1,2}, \ldots, n_{h_{K_{i,j}}+1,K_{i,j}}$ all refer to the destination node *j* (see Fig. 2). We use this notation to ease our presentation in next sections. Here, we define $\mathcal{S}_{i,j} = \bigcup_{k=1}^{K_{i,j}} P_k$ as the set of constituent nodes of all $K_{i,j}$ node-disjoint routes between the source node *i* and the destination node *j*.

Without loss of generality, we assume the criteria for finding routes is minimizing the hop-count. Accordingly, the *k*th node-disjoint route between two nodes (i.e., P_k) is the route whose rank is *k*. Here, we do not consider any preference in ranking of routes with the same number of hops. That is, if two routes have the same number of hops, one of them is ranked after the other one. It is also worthwhile to mention that if the source and destination are neighbors, then the first ranked route P_1 will be the direct link between the two nodes (single-hop route). In such a case, no alternative route is required for communication, because the two nodes will communicate through the direct link between them until one of them fails. Failure of the source or destination will end their communication. In our analysis in this paper, we distinguish between single-hop and multi-hop routes (connections).

2.3. Medium access control mechanism

We consider two types of MAC protocols: protocols which support ARQ, and protocols which do not support ARQ. If ARQ is supported, an acknowledgment (ACK) is transmitted by the receiving node for each data packet that is received error-free over the physical link (see Fig. 3). The sending node will retransmit the packet, if no ACK is received. This continues until the sender receives an ACK for the packet, or the maximum number of transmission tries, *M*, is reached. Therefore, a data packet or its ACK might be transmitted $m \leqslant M$ times. If, however, ARQ is not supported, the packet is sent only once and no ACK is transmitted. MAC protocol in IEEE 802.11b/g/n standards supports mandatory ARQ, while in IEEE 802.15.4 standard support for ARQ is optional. In IEEE 802.15.4, MAC header of each transmitted packet indicates whether the receiver needs to acknowledge the packet or not. Thus, depending on the link technology deployed in a wireless ad hoc network, ARQ may or may not be supported. For this reason, we consider both type of MAC protocols in this work.

The analysis we provide here is independent of the type of MAC mechanism. Nevertheless, since wireless ad hoc networks are autonomous systems without a central controller, usually carrier sense multiple access (CSMA) mechanisms are deployed. Here, we only make the following assumptions with regard to the MAC mechanism:

(1) Transmission time of data packets over wireless links is assumed to be negligible compared to the inter-arrival time of data packets – *belonging to the same session* – from the application layer. Transmission time includes waiting time at the MAC layer to access the channel as well as the duration required to transmit all bits of the packet on the physical link.

[1] Here, we basically assumed that a reactive route discovery mechanism is deployed in the ad hoc network. Reactive protocols are shown to be more effective in these networks.

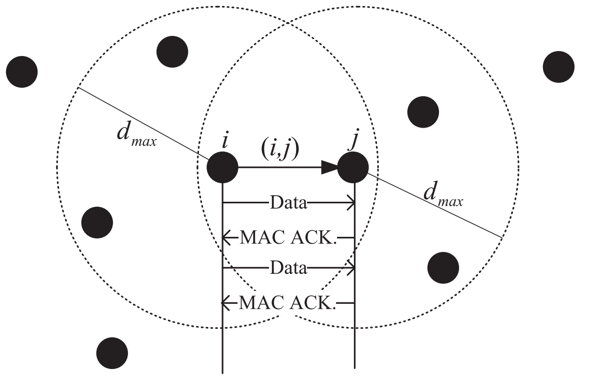

Fig. 3. Transmission of data packets and MAC acknowledgments over a physical link.

In fact, we assume MAC mechanism is efficient enough to achieve relatively a small channel access waiting time. Furthermore, wireless technology supports relatively a high data rate such that transmission time becomes negligible compared to inter-arrival time of packets – *belonging to the same session* – from the application layer.

(2) When an intermediate node relays a packet, the time required to receive and forward the packet is negligible compared to the inter-arrival time of data packets – *belonging to the same session* – to that intermediate node. That is, in addition to having a negligible transmission time, the processing time of a packet at an intermediate node is also negligible compared to the inter-arrival time of packets – *belonging to the same session*.

(3) When ARQ is supported, we also assume that the total time required to deliver a packet over a physical link to the receiver is negligible compared to the inter-arrival of packets – *belonging to the same session* – to the MAC layer of the source node (from application layer) or the MAC layer of relay nodes (from physical layer).

We emphasize again that the transmission delay and the processing time of packets have been considered negligible compared to the inter-arrival time of packets which belong to the same session between the source and the destination. In other words, we assume that the underlying network layer could transmit a packet to the next hop before the next packet generated by the source node arrives at an intermediate node. Note that with the increase in data rates of wireless technologies, efficiency of data compression techniques, and increase in the processing speed of small processors such assumptions are not very unrealistic. We made these assumptions to reduce complexity of calculating the energy consumption rate of intermediate nodes when they forward packets of the source node hop-by-hop to the destination node.

2.4. Packet reception

In accordance with wireless standards such as IEEE 802.11 and IEEE 802.15.4, we assume a packet transmitted on a physical link consists of three parts: a preamble for transmitter–receiver synchronization, a header specifying the payload length, and the payload. The payload includes higher layer headers (e.g., MAC, IP, TCP/UDP) as well as the application data. The payload is protected by cyclic redundancy check (CRC). The packet length is referred to as the length of the entire packet including the header and the preamble.

There could be four scenarios in which a packet transmitted on a physical link is lost. First, if the received power of signals carrying information of the preamble is lower than the threshold required for detection (e.g., due to high interference power), then the packet will not be received at all. Second, if the receiver detects the preamble with error, then it will not continue to receive the header and the payload. Thus, the packet is lost. Third, when the preamble is detected error-free but the header is erroneous, the receiver stops receiving the payload. Forth, if both preamble and header are received error-free but the payload fails to pass the CRC, then the packet will be discarded. In the first scenario, a lost packet is not received at all. In the second and third scenarios, a lost packet is received partially. Nevertheless, in the fourth scenario, a lost packet is received completely. Since the size of the preamble and the header are usually much smaller than the size of the payload, the probability that a lost packet is received completely is much higher than the probability that it is received partially. Therefore, we assume that the receiver likely consumes the same amount of energy for receiving lost (corrupted) packets compared to receiving error-free packets.

Here, we define $p(u,v)$ as the probability of error-free reception of a data packet of size L bits transmitted by node u to node v over the physical link (u,v). $\mathbb{P} = \{p(u,v)\}$ is defined as the set of these values for all links in the network. When ARQ is supported, we also define $\mathbb{Q} = \{q(u,v)\}$, in which $q(u,v)$ is probability of error-free reception of an ACK of size L_a bits acknowledging the reception of the data packet which has been transmitted over (u,v). We emphasize that $p(u,v)$ is the probability of error-free reception of data packets considering the effect of transmission error due to fading and shadowing on wireless channels as well as the effect of collision due to strong interference. The same is true for $q(u,v)$.

3. Node-to-node communication lifetime: problem statement and formulation

We model and analyze node-to-node communication lifetime as defined in the sequel.

3.1. Problem statement

Assume an arbitrary source node $i \in \mathbb{V}$ transmits packets to an arbitrary destination node $j \in \mathbb{V}$ with the rate λ packet/s. The packet transmission is started at the network startup (without loss of generality). At first, P_1, the first node-disjoint route between the source node and the destination node, is used to transfer packets. If P_1 fails due to failure of its intermediate nodes, P_2 is used provided that the source and the destination both are still alive. This continues until either the source or the destination or the last available route between them, $P_{K_{i,j}}$ fails.

Our goal in this work is to analyze the maximum duration that two nodes in a wireless ad hoc network can communicate with each other. To this end, nodes belonging to $\mathcal{S}_{i,j}$ must only carry and be affected by the generated traffic by node i destined to node j. To clarify this, assume that some nodes of $\mathcal{S}_{i,j}$ carry traffic other than the traffic generated by i. They may act as intermediate nodes or even source or destination in other traffics. Even if these nodes are not directly involved in other traffics, they may overhear packets belonging to other traffics. In any case, their energy consumption will increase compared to the case that they are not involved with or affected by other traffics in the network. If energy consumption rate of such nodes belonging to $\mathcal{S}_{i,j}$ increases, they live for a shorter time. This, in turn, may reduce the duration that i can communicate to j via nodes in $\mathcal{S}_{i,j}$. The duration that i can transfer packets to j, denoted by $T_{i,j}$, is maximized if nodes in $\mathcal{S}_{i,j}$ do not carry other traffics or are not affected by other traffics in the network. $T_{i,j}$ as defined here is the *maximum* duration that i can transfer packets to j.

3.2. Problem formulation

In general, a wireless ad hoc network might be a disconnected network. Thus, there may not be a route between two nodes which are outside each other's transmission range. In such a case their communication lifetime is basically zero. However, if there is at least one route between two nodes or the two nodes are neighbors, their communication lifetime is either the lifetime of the source node, the destination node, or the last available route between them $P_{K_{i,j}}$.

We first assume that nodes are not neighbors. Let T_k be the time (with respect to the network start-up $t = 0$) at which P_k fails, and the use of the next route, P_{k+1} is started for packet transfer from node i to node j. Furthermore, let $c(n_{l,k})$ (J/s) be the energy consumption rate of $n_{l,k} \in P_k$ when P_k is in-use for packet transfer. We also denote $B_{n_{l,k}}(T_{k-1})$ as the residual battery energy of $n_{l,k} \in P_k$ at the time of failure of P_{k-1}. Given these notations, we can determine T_1 as follows:

$$T_1 = \min(t_{1,1}, t_{2,1}, \ldots, t_{l,1}, \ldots, t_{h_1,1}, t_{h_1+1,1}),$$

in which $t_{l,1}$ is defined as

$$t_{l,1} = \frac{B_{n_{l,1}}(0)}{c(n_{l,1})}, \quad \forall l = 1..h_1 + 1.$$

Here, there could be several possibilities. If $T_1 = t_{1,1}$, failure of the source node i ends the communication between the source and the destination node. If $T_1 = t_{h_1+1,1}$, failure of the destination node j ends the communication. If neither $T_1 = t_{1,1}$ nor $T_1 = t_{h_1+1,1}$, but there is only one route between i and j (i.e., $K_{i,j} = 1$), the communication ends due to lack of alternative routes between i and j. If there is a second route to continue the communication after failure of the first route, we can calculate the lifetime of the second route as follows:

$$T_2 = \min(t_{1,2}, t_{2,2}, \ldots, t_{l,2}, \ldots, t_{h_2,2}, t_{h_2+1,2}),$$

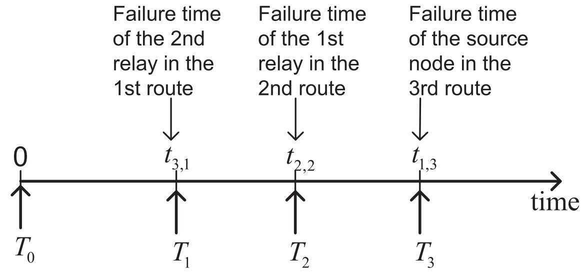

Fig. 4. Illustrating of the communication lifetime of two nodes for the example shown in Fig. 2. Failure of the first route happens at the time that the second relay fails (i.e., $t_{3,1}$), and failure of the second route happens at the time that the first relay fails (i.e., $t_{2,2}$). Failure of the third route happens at the time that the source node fails (i.e., $t_{1,3}$).

where $t_{l,2}$ is defined as

$$t_{l,2} = T_1 + \frac{B_{n_{l,2}}(T_1)}{c(n_{l,2})}, \quad \forall l = 1..h_2 + 1.$$

We check again whether the source node i or the destination node j has failed, or they are still alive, but the second route has failed. According to our notation, in either case $T_{i,j} = T_2$. If the source and the destination are still alive, and there is another route to continue the communication, this procedure is repeated until the communication fails (see Fig. 4). In general, we have

$$T_k = \min(t_{1,k}, t_{2,k}, \ldots, t_{l,k}, \ldots, t_{h_k,k}, t_{h_k+1,k}), \tag{1}$$

in which

$$t_{l,k} = T_{k-1} + \frac{B_{n_{l,k}}(T_{k-1})}{c(n_{l,k})}, \quad \forall l = 1..h_k + 1, \ k = 1 \ldots K_{i,j}. \tag{2}$$

Now, assume that the two nodes i and j are neighbors. That is, the single-hop route $P_1 = \{n_{1,1} = i, n_{h_1+1,1} = j\}$ is used for communication between them. In such a case, we obviously have

$$T_{i,j} = T_1 = \min(t_{1,1}, t_{h_1+1,1}).$$

As we see, to calculate the communication lifetime of two nodes, we need to determine the energy consumption rate of nodes, $c(n_{l,k})$, and their remaining battery energy $B_{n_{l,k}}(T_{k-1})$. In the next two sections, we model these values in networks with and without ARQ support. On the basis of this modeling and the problem formulation presented in this section, we will present in the next two sections numerical algorithms to calculate the node-to-node communication lifetime between two specific nodes.

4. Energy consumption and remaining battery energy of nodes without ARQ

We first present a model for calculating the energy consumed by a transmitting and a receiving node to exchange a packet over a physical link. On the basis of this model, we then determine the energy consumption rate of nodes.

4.1. Consumed energy for packet exchange over a physical link

Similar to [13,14], we assume that the consumed energy for transmission and reception of a packet over a physical link increases linearly with the packet size. When ARQ is not supported, every packet is transmitted once. If we denote e_t and e_r as the energy consumed to transmit and receive a packet of length L bits over a physical link, we respectively have

$$\begin{cases} e_t = \epsilon_t L, \ \forall(u,v) \in \mathbb{E}, \\ e_r = \epsilon_r L, \ \forall(u,v) \in \mathbb{E}, \end{cases} \tag{3}$$

where, ϵ_t and ϵ_r (J/bit) are the energy consumed to transmit and receive a single bit of the packet, respectively.

In addition to the energy consumed by nodes for transmission and reception of data packets, there are some other sources of energy consumption for nodes. For instance, in wireless ad hoc networks, nodes consume energy for transmission and reception of control packets for routing and neighbor discovery (background traffic) and for channel sensing. Even when nodes do not send or receive any control or data packet, they may also consume energy for non-communication purposes such as application execution. To take into account these sources of energy consumption in our analysis, we abstract them for each node u by a parameter $f(u)$ (J/s). Set of these values for all nodes in the network are denoted by $\mathbb{F} = \{f(u)\}_{u=1}^{N}$. We emphasize that the amount of $f(u)$ is assumed to be different for different nodes. One reason is, different nodes might have different number of neighboring nodes broadcasting beacons and other control messages. Furthermore, in a wireless ad hoc network, nodes might be heterogeneous and consume different amount of energy for non-communication purposes. Defining $f(u)$ in this general form makes our analysis also general and independent of the type of networking protocols deployed. We refer to $f(u)$ as *idle-mode energy consumption rate* of node u. Here, we say a node is at the idle mode when it does not transmit or receive any *data packet which belongs to the active connection between a source and a destination node*. This should not be mistaken by energy consumption of transceivers when they are idle and do not transmit or receive any packet (data or control).

4.2. Energy consumption rate of nodes

A single hop route is always the last route which is used for packet transfer, because failure of the source or the destination will end the communication. If the source node transmits data packets to the destination node with the rate λ packets/s, their energy consumption rate in a single-hop route is as follows:

$$\begin{cases} c(n_{1,k}) = \lambda e_t + f(n_{1,h_k}), \\ c(n_{h_k+1,k}) = \lambda e_r + f(n_{h_k+1,k}), \end{cases} \tag{4}$$

where e_t and e_r are as defined in (3). The following remark is the result of (4):

Remark 1. Assuming $f(n_{1,h_k}) = f(n_{h_k+1,k})$, the energy consumption rate of the source node in a single-hop route when ARQ is not supported is greater than the energy consumption rate of the destination node, if $\epsilon_t > \epsilon_r$. Otherwise, the energy consumption rate of the destination node is higher.

In a multi-hop route, each relay node forwards data packets that it receives from the previous relay node. There are two issues which must be considered for computing the energy consumption rate of nodes. First, the packet forwarding rate in each hop depends on the reliability of the previous links along the route. When $P_k = \langle n_{1,k}, n_{2,k}, \ldots, n_{h_k,k}, n_{h_k+1,k}\rangle$ is used to transfer data packets from the source node to the destination node, $n_{l,k} \in P_k$ forwards packets with the rate $\lambda(n_{l,k})$, where

$$\lambda(n_{l,k}) = \begin{cases} \lambda, \ l = 1, \\ \lambda \prod_{m=1}^{l-1} p(n_{m,k}, n_{m+1,k}), \quad \forall l = 2..h_k, \end{cases}$$

and λ is the packet transmission rate of the source node.

Remark 2. Due to packet loss over physical links, the packet forwarding rate of each relay node is lower than or equal to the packet forwarding rate of its upstream nodes. That is, $\lambda(n_{l,k}) \leqslant \lambda(n_{l-1,k}) \leqslant \cdots \leqslant \lambda(n_{1,k})$.

The second issue that we must consider is that each node overhears packets transmitted by its downstream node as shown in Fig. 5. The figure shows that, for example, the second relay overhears packets transmitted by the third relay. Nevertheless, the third relay (in general the last relay), which is next to the destination, does not overhear any packet. Taking into account these two issues and assumptions in Section 2.3, the energy consumption rates of nodes in a multi-hop route are computed as follows:

$$\begin{cases} c(n_{1,k}) = \lambda(n_{1,k})e_t + \lambda(n_{2,k})e_r + f(n_{1,k}), \\ c(n_{l,k}) = \lambda(n_{l-1,k})e_r + \lambda(n_{l,k})e_t + \lambda(n_{l+1,k})e_r + f(n_{l,k}), \\ \quad \forall l = 2..h_k - 1, \\ c(n_{h_k,k}) = \lambda(n_{h_k-1,k})e_r + \lambda(n_{h_k,k})e_t + f(n_{h_k,k}), \\ c(n_{h_k+1,k}) = \lambda(n_{h_k,k})e_r + f(n_{h_k,k}). \end{cases} \tag{5}$$

Remark 2 implies that $\lambda(n_{1,k}) \geqslant \lambda(n_{h_k,k})$, which means if $f(n_{1,k}) \geqslant f(n_{h_k,k})$, then $c(n_{1,k}) > c(n_{h_k+1,k})$, because $\epsilon_t > 0$. Furthermore, $\lambda(n_{l,k}) \geqslant \lambda(n_{l+1,k})$, $\forall\ l = 2..h_k$. According to (5), if $f(n_{l,k}) \geqslant f(n_{l+1,k})$, we have $c(n_{l_k}) > c(n_{l+1,k})$, $\forall l = 2..h_k$. Thus, we can have the following two remarks.

Remark 3. Assuming $f(i) \geqslant f(j)$, the energy consumption rate of the source node in a multi-hop route when ARQ is not supported is always greater than the energy consumption rate of the destination node.

Remark 4. Assuming $f(n_{l,k}) \geqslant f(n_{l+1,k})$, $\forall l = 2..h_k$, when ARQ is not supported, the energy consumption rate of each relay node in a multi-hop route is always greater than the energy consumption rate of its downstream relay node.

4.3. Remaining battery energy of nodes

In this section, we determine the remaining battery energy of nodes in a multi-hop route P_k at the time that this route is used for packet transfer to the destination node. When previous routes, $P_1, P_2, \ldots, P_{k-1}$ have been in use, the source node has continuously transmitted packets and the destination node has continuously received pack-

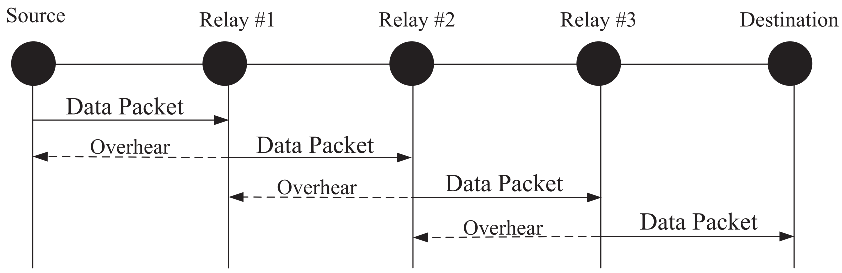

Fig. 5. Each node in a multi-hop connection overhears packets transmitted by its downstream node.

ets. Thus, their remaining battery energy at the time that P_k is used for packet transfer, T_{k-1}, is computed as,

$$\begin{cases} B_{n_{1,k}}(T_{k-1}) = B_{n_{1,k}}(0) - \sum_{d=1}^{k-1} c(n_{1,d})(T_d - T_{d-1}), \\ B_{n_{h_k+1,k}}(T_{k-1}) = B_{n_{h_k+1,k}}(0) - \sum_{d=1}^{k-1} c(n_{h_k+1,d})(T_d - T_{d-1}). \end{cases} \quad (6)$$

To determine the remaining battery energy of relay nodes in a multi-hop route, we notice that the relay nodes in this route may overhear packets transmitted by nodes in the previously in-use routes. That is, while $P_1, P_2, \ldots, P_{k-1}$ have been used, relay nodes in P_k may overhear packets transmitted along these routes (see Fig. 6). To consider this phenomenon in our analysis, we assume $\gamma_{l,k}(d)$ is the number of nodes in P_d, $\forall d = 1..k-1$ – including the source and the destination nodes – which are neighboring nodes of $n_{l,k} \in P_k$. We define $\Phi_{l,k}(d) = \{\varphi_1, .., \varphi_{\gamma_{l,k}(d)}\}$ as the set of such nodes. With this definition, we have,

$$B_{n_{l,k}}(T_{k-1}) = B_{n_{l,k}}(0) - f(n_{l,k})T_{k-1} - \sum_{d=1}^{k-1} \sum_{\varphi_i \in \Phi_{l,k}(d)} (T_d - T_{d-1})\lambda(\varphi_i)e_r, \quad \forall l = 2..h_k, \; k = 1..K_{i,j} \quad (7)$$

where, $\lambda(\varphi_i)$ is the rate that φ_i forwards packets, when it was part of a route. The following theorem specifies the maximum value of $\gamma_{l,k}(d)$.

Theorem 1. *Let $\{P_k\}_{k=1}^{K}$ be the ordered set of node-disjoint routes between two nodes with regard to the hop-count. Each relay node in P_k can have physical link with at most three nodes in P_d, including the source and the destination nodes, $\forall l = 2..h_k$ and $\forall k, d < k$.*

Proof. Let $\langle A, B, C, D \rangle$ in Fig. 7 be the minimum-hop route between nodes A and D in the network, and E is an arbitrary node (see Fig. 7 as an illustrative example). Node E could have links with A, B, and C, without violating the assumption that $\langle A, B, C, D \rangle$ is the minimum-hop path. Nevertheless, if there is a link between E and D (i.e., E and D are neighbors), then the minimum-hop route from A to D will be $\langle A, E, D \rangle$. This violates the initial assumption. □

Corollary 1. *If the discovered routes are minimum-hop routes, then $0 \leqslant \gamma_{l,k}(d) \leqslant 3$, $\forall l = 2..h_k$ and $\forall k, d < k$.*

Corollary 2. *Since the first relay node is a neighboring node of the source node and the last relay node is a neighboring node of the destination node, we have $1 \leqslant \gamma_{l,k}(d) \leqslant 3$, for $l \in \{2, h_k\}$ and $\forall k, d < k$.*

So far, we calculated the energy consumption rate of nodes and their remaining battery energy in single-hop and multi-hop routes when ARQ is not supported. We can use (4) to calculate the energy consumption rate of the source and the destination node in a single-hop route,

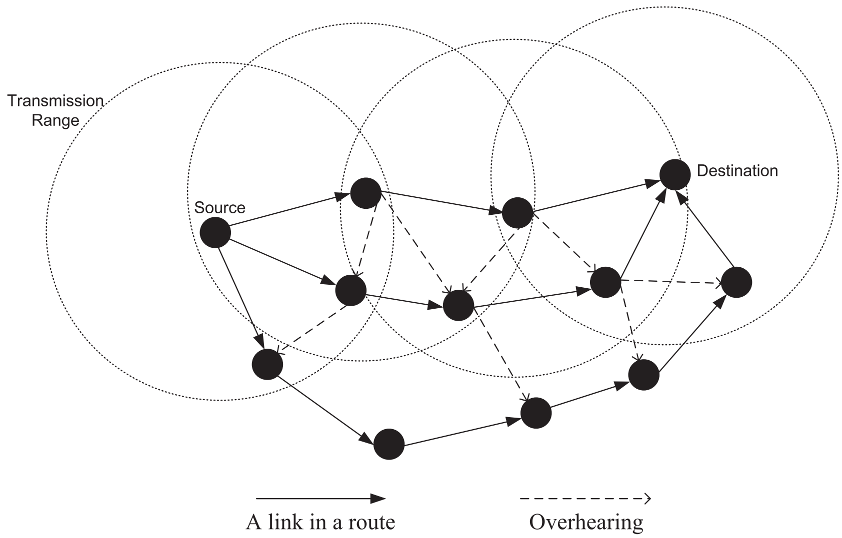

Fig. 6. Relay nodes in other routes may overhear packets transmitted along an in-use route.

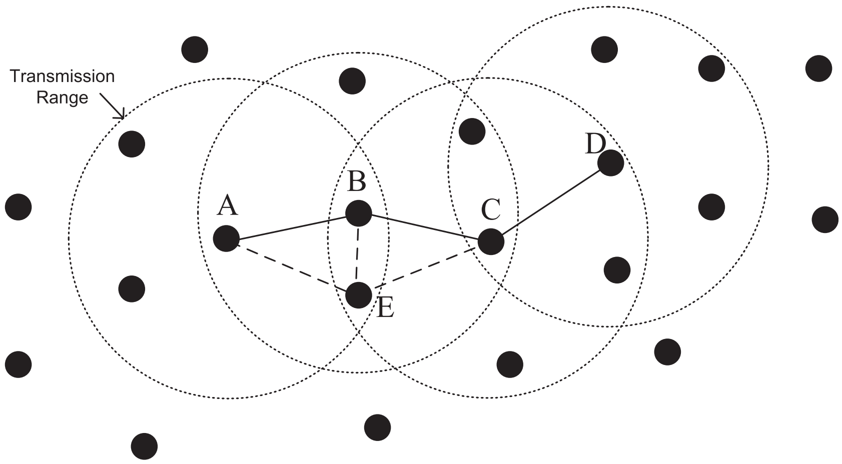

Fig. 7. Illustration of Theorem 1. Note that the circular transmission range in this figure (as well as in Figs. 6 and 3) is only for the sake of illustration. The analytical model and the analysis in this paper is not based on circular transmission range. Our model only considers a logical neighborhood relation between nodes.

and (5) to calculate the energy consumption rate of the source node, the destination node, and relay nodes in a multi-hop route. On the basis of the formulation we provided in this section and in the previous section, we can derive an algorithm to calculate the communication lifetime of two specific nodes *i* and *j* in the network. The algorithm has been presented in Algorithm 1 and in its related procedure presented in Algorithm 2.

To determine the communication lifetime of two nodes using Algorithm 1, we need to know the network topology, packet delivery ratio of links, idle-mode energy consumption rate of nodes, and their battery energy at the moment that we want to determine the communication lifetime between the two nodes. In practice, we need some efficient protocols for gathering this information and transferring them to a monitoring room. Then, at the monitoring room operators could monitor communication lifetime of the active connections in the network. Design of such efficient protocols needs further investigation in future work.

Algorithm 1. Connection lifetime of two nodes *i* and *j*.

INPUT $\mathbb{G}(\mathbb{V},\mathbb{E})$, $\mathbb{B}(0)$, $\mathbb{F}$, $\mathbb{P}$, ϵ_r, ϵ_t, λ, $\mathbb{Q}$ (When ARQ is supported)
$k = 1$
$T_{k-1} \leftarrow 0$
loop
  Find P_k
  if $P_k = \emptyset$ **then**
    return T_{k-1}
  end if
  Calculate $c(n_{l,k})$, $\forall l = 1 \ldots h_k + 1$
  $t_{l,k} \leftarrow T_{k-1} + \frac{B_{n_{l,k}}(T_{k-1})}{c(n_{l,k})}$
  $T_k \leftarrow \min(t_{1,k}, t_{2,k}, \ldots, t_{h_k,k}, t_{h_k+1,k})$
  if $T_k = \min(t_{1,k}, t_{h_k+1,k})$ **then**
    return T_k
  end if
  Update $\mathbb{B}$ and $\mathbb{G}$
  $k \leftarrow k + 1$
end loop

Algorithm 2. Procedure update $\mathbb{B}$ and $\mathbb{G}$ when ARQ is not supported.

$B_{n_{1,k}} \leftarrow B_{n_{1,k}} - C(n_{l,k})(T_k - T_{k-1})$
$B_{n_{h_k+1,k}} \leftarrow B_{n_{h_k+1,k}} - C(n_{h_k+1,k})(T_k - T_{k-1})$
for $u = 1$ to N & $u \notin P_k$ **do**
  $B_u \leftarrow B_u - f(u)(T_k - T_{k-1})$
  for $l = 1$ to h_k **do**
    if $(u, n_{l,k}) \in \mathbb{E}$ **then**
      $B_u \leftarrow B_u - \lambda(n_{l,k})\epsilon_r L(T_k - T_{k-1})$
    end if
  end for
  if $B_u < 0$ **then**
    Remove u from $\mathbb{G}$
  end if
end for
Remove $P_k - \{n_{1,k}, n_{h_k+1,k}\}$ from $\mathbb{G}$

5. Energy consumption and remaining battery energy of nodes with ARQ

In this section, we model the energy consumption rate and the remaining battery energy of nodes when ARQ is supported. To this end, we first present a model to determine the total energy consumed to exchange a packet over a physical link when ARQ is supported.

5.1. Consumed energy for packet exchange over a physical link

When ARQ is supported, a data packet and its corresponding ACK may be transmitted several times over a physical link. In this case, the energy consumed to exchange a packet over the physical link includes the energy consumed for retransmission of the data packet as well as the energy consumed for (re)transmission and (re)reception of ACKs. Thus, the expected energy consumed by u and v when u tries to deliver a data packet of length L bits to v is

$$\begin{cases} e_t(u,v) = a(u,v)\epsilon_t L + b(u,v)\epsilon_r L_a, \\ e_r(u,v) = a(u,v)\epsilon_r L + b(u,v)\epsilon_t L_a, \end{cases} \quad (8)$$

where $a(u,v)$ is the expected number of transmission tries of node u to deliver a packet to node v, and $b(u,v)$ is the expected number of ACKs sent by v to u for a data packet. Here, L_a is the length of the ACK packet in bits. Obviously, $L \geqslant L_a$, because L includes higher layers header and user data, while L_a only includes MAC and physical layer headers.

Intuitively, we can state that $a(u,v) \geqslant b(u,v), \forall (u,v)$. The reason is, an ACK is sent when a data packet is received possibly after several tries. Hence, from (8), we can conclude that if $\epsilon_t \geqslant \epsilon_r$, $e_t(u,v) \geqslant e_r(u,v)$. Otherwise, $e_t(u,v) < e_r(u,v)$. Therefore, we can state the following remark:

Remark 5. When ARQ is supported, the consumed energy by a transmitting node to deliver a packet over a physical link is greater than the energy consumed by the receiving node, if $\epsilon_t \geqslant \epsilon_r$. Otherwise, the energy consumed by the receiving node will be greater.

As a matter of fact, $a(u,v)$ and $b(u,v)$ depend on the quality of the forward link (u,v) and the reverse (v,u). The significance of Remark 5 is that the energy consumed by a transmitting node is always greater than the energy consumed energy by the receiving node, regardless of the quality of the link between them.

To calculate $a(u,v)$ and $b(u,v)$, we notice that a packet will be transmitted x times, if the packet itself or its ACK is lost in the last $x-1$ transmission tries. If random variable $\mathcal{X}$ denotes the number of transmission tries to deliver a data packet over a physical link, we have

$$\Pr\{\mathcal{X} = x\} = \begin{cases} (1-pq)^{x-1}pq, & \forall x = 1..M-1, \\ (1-pq)^{M-1}, & x = M, \end{cases} \quad (9)$$

where $p = p(u,v)$ is the probability of error-free reception of a data packet of length L bits transmitted over (u,v), and $q = q(u,v)$ is the probability of error-free reception of its ACK of length L_a transmitted over (v,u). Furthermore, if the random variable $\mathcal{Y}$ denotes the number of ACKs transmitted for the data packet, we have proved in Appendix that,

$$\Pr\{\mathcal{Y}=x\} = \begin{cases} (1-p)^M, \; x=0; \\ \binom{M-1}{x-1} p^x (1-q)^{x-1}(1-p)^{M-x} \\ \quad + \binom{M-1}{x} p^x (1-q)^x (1-p)^{M-x} \\ \quad + \sum_{m=x}^{M-1} \binom{m-1}{x-1} p^x (1-q)^{x-1} (1-p)^{m-x} q, \; \forall x = 1..M-1; \\ p^M (1-q)^{M-1}, \; x = M. \end{cases} \quad (10)$$

We can compute the exact value of $a(u,v)$ and $b(u,v)$ for any value of M using the probability density functions of $\mathcal{X}$ and $\mathcal{Y}$ given in (9) and (10), respectively. Nevertheless, when there is no limitation on the number of retransmissions over a physical link ($M \to \infty$), a data packet will be transmitted until the packet itself and its acknowledgment is received correctly. In such a case, we have $a(u,v) = 1/(p(u,v)q(u,v))$. Furthermore, since a data packet can be retransmitted as many times as required to receive it correctly, $b(u,v)$ will not depend on $p(u,v)$ anymore. Hence, the expected number of transmission tries to receive an ACK correctly will be $b(u,v) = 1/q(u,v)$. As a result, the following inequalities will be always true regardless of the value of M,

$$\begin{cases} a(u,v) \leqslant \frac{1}{p(u,v)q(u,v)}, \; \forall M > 0, \\ b(u,v) \leqslant \frac{1}{q(u,v)}, \; \forall M > 0. \end{cases}$$

It is also worthwhile to mention that if $p(u,v) \to 1$, and $q(u,v) \to 1$, then $a(u,v) \to 1$, and $b(u,v) \to 1$, $\forall M > 0$.

5.2. Energy consumption rate of nodes

Knowing the consumed energy for packet exchange over a physical link, we can determine the energy consumption rate of nodes in single-hop and multi-top routes. Considering the assumptions in Section 2.3, the energy consumption rate of the source and destination node in a single-hop route is as follows:

$$\begin{cases} c(n_{1,k}) = \lambda e_t(n_{1,k}, n_{h_k+1,k}) + f(n_{1,k}), \\ c(n_{h_k+1,k}) = \lambda e_r(n_{1,k}, n_{h_k+1,k}) + f(n_{h_k+1,k}), \end{cases} \quad (11)$$

where $e_t(n_{1,k}, n_{h_k+1,k})$ and $e_r(n_{1,k}, n_{h_k+1,k})$ are as expressed in (8).

To determine the energy consumption rate of nodes in multi-hop routes, we first determine the packet forwarding rate of nodes when ARQ is supported. When $P_k = \langle n_{1,k}, n_{2,k}, \ldots, n_{h_k,k}, n_{h_k+1,k} \rangle$ is used to transfer data packets, $n_{l,k}$ will forward packets at a rate

$$\lambda(n_{l,k}) = \begin{cases} \lambda, & l = 1 \\ \lambda \prod_{m=1}^{l-1} R(n_{m,k}, n_{m+1,k}), & \forall l = 2..h_k. \end{cases} \quad (12)$$

where $R(n_{m,k}, n_{m+1,k})$ is reliability of $(n_{m,k}, n_{m+1,k})$ link in P_k defined as

$$R(n_{m,k}, n_{m+1,k}) = 1 - \left[1 - p(n_{m,k}, n_{m+1,k})\right]^M.$$

When ARQ is supported, nodes along a route may overhear not only transmitted data packets by their downstream nodes, but also transmitted ACKs by their upstream nodes (see Fig. 8). In the example shown in the figure, the second relay overhears packets transmitted by the third relay to the destination node, and ACKs sent by the first relay to the source node. In general, depending on the number of relay nodes in a route and their position, a relay node may overhear nothing or it may overhear either data packets, ACKs, or both of them. In a route with only one relay node, the relay overhears nothing. In a route with two relays, the first relay overhears data packets transmitted by the second relay to the destination, and

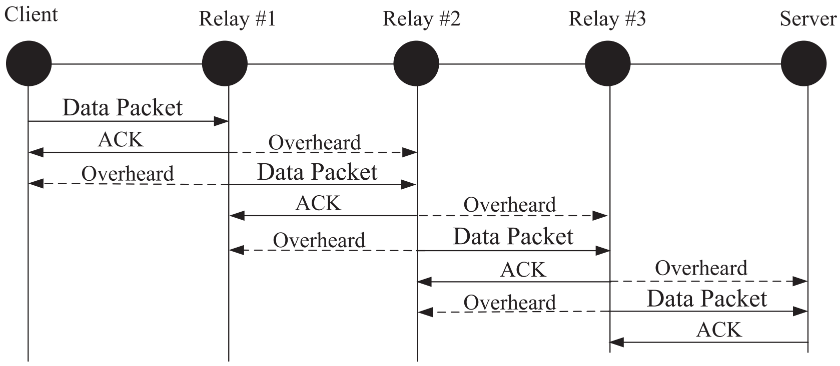

Fig. 8. When ARQ is supported nodes along a route may overhear transmitted data packets by their downstream nodes and transmitted ACKs by their upstream nodes.

the second relay overhears ACKs sent by the first relay to the source node. In a route with more than two relays the first relay only overhears data packets, the last relay only overhears ACKs, and other relays overhears both ACKs and data packets.

Considering the above explanation, we can compute the energy consumption rate of the source and the destination nodes in a multi-hop route, when P_k is in-use, as follows:

$$\begin{cases} c(n_{1,k}) = \lambda(n_{1,k})e_t(n_{1,k},n_{2,k}) + \lambda(n_{2,k})a(n_{2,k},n_{3,k})\epsilon_r L + f(n_{1,k}), \\ c(n_{h_k+1,k}) = \lambda(n_{h_k,k})e_r(n_{h_k,k},n_{h_k+1,k}) + \lambda(n_{h_k-1,k})b(n_{h_k-1,k},n_{h_k,k})\epsilon_r L_a \\ \qquad + f(n_{h_k+1}). \end{cases} \tag{13}$$

Remark 6. Due to the effect of quality of links, the energy consumption rate of the source node $n_{1,k}$ in a multi-hop route could be smaller than the energy consumption rate of the destination node n_{h_k+1}, even if $f(n_{1,k}) > f(n_{h_k+1})$.

As mentioned before, the energy consumption rate of a relay node in a multi-hop route depends on the hop-count of the route and the position of the relay in the route. In a route with two hops, $P_k = \langle n_{1,k}, n_{2,k}, n_{3,k}\rangle$, the energy consumption rate of the relay node is

$$c(n_{2,k}) = \lambda(n_{1,k})e_r(n_{1,k},n_{2,k}) + \lambda(n_{2,k})e_t(n_{2,k},n_{3,k}) + f(n_{2,k}). \tag{14}$$

In a route with three hops, $P_k = \langle n_{1,k}, n_{2,k}, n_{3,k}, n_{4,k}\rangle$, the energy consumption rate of relay nodes $n_{2,k}$ and $n_{3,k}$ is as follows:

$$\begin{cases} c(n_{2,k}) = \lambda(n_{1,k})e_r(n_{1,k},n_{2,k}) + \lambda(n_{2,k})e_t(n_{2,k},n_{3,k}) \\ \qquad + \lambda(n_{3,k})a(n_{3,k},n_{4,k})\epsilon_r L + f(n_{2,k}), \\ c(n_{3,k}) = \lambda(n_{2,k})b(n_{1,k},n_{2,k})\epsilon_r L_a + \lambda(n_{2,k})e_r(n_{2,k},n_{3,k}) \\ \qquad + \lambda(n_{3,k})e_t(n_{3,k},n_{4,k}) + f(n_{3,k}). \end{cases} \tag{15}$$

In a route with more than three hops, $P_k = \langle n_{1,k}, \ldots, n_{l,k}, \ldots, n_{h_k+1,k}\rangle$, the energy consumption rate of the first relay, $n_{2,k}$, and the last relay, $n_{h_k,k}$, is the same as the energy consumption rate of the first and the second relay node in a route with three hops, respectively. The energy consumption rate of other relay nodes is as follows:

$$\begin{aligned} c(n_{l,k}) = {}& \lambda(n_{l-2,k})b(n_{l-2,k},n_{l-1,k})\epsilon_r L_a + \lambda(n_{l-1,k})e_r(n_{l-1,k},n_{l,k}) \\ & + \lambda(n_{l,k})e_t(n_{l,k},n_{l+1,k}) + \lambda(n_{l+1,k})a(n_{l+1,k},n_{l+2,k})\epsilon_r L \\ & + f(n_{l,k}), \quad \forall l = 3..h_k - 1. \end{aligned} \tag{16}$$

The fact that the energy consumption rate of which node in a multi-hop route is the largest depends on the quality of links in that path.

5.3. Remaining battery energy of nodes

In this section, we define the remaining battery energy of nodes when a route is used for packet transfer (i.e., $B_{n_{l,k}}(T_{k-1})$). Similar to the case that ARQ is not supported, the remaining battery energy of the source and the destination nodes, when the source node switches from P_{k-1} to P_k, is defined by (6). To determine the residual battery energy of relay nodes in P_k, we notice that when ARQ is supported, in addition to the data packets, nodes in other remaining routes may overhear ACKs sent by nodes of an in-use route as well. Nevertheless, a node may overhear ACKs and data packets from different set of nodes, because the source node never transmits an ACK and the destination node never transmits a data packet. To formulate this, we define $\Phi_{l,k}(d) = \{\varphi_1, .., \varphi_{\gamma_{l,k}(d)}\}$ as the set of nodes in $P_d - \{n_{h_d+1,d}\}$ ($n_{h_d+1,d}$ is the destination node), which are neighboring nodes of $n_{l,k} \in P_k$, where $\gamma_{l,k}(d)$ is the size of this set. The transmitted data packets by nodes of $\Phi_l(k,d)$ will be overheard by $n_{l,k} \in P_k$. We also define $\Delta_{l,k}(d) = \{\delta_1, .., \delta_{\gamma'_{l,k}(d)}\}$ as the set of nodes in $P_d - \{n_{1,d}\}$ ($n_{1,d}$ is the source node), which are neighboring nodes of $n_{l,k} \in P_k$, where $\gamma'_{l,k}(d)$ is the size of this set. The transmitted ACKs by these nodes will be overheard by $n_{l,k} \in P_k$. Given these two sets, $B_{n_{l,k}}(T_{k-1})$ is obtained as

$$\begin{aligned} B_{n_{l,k}}(T_{k-1}) = {}& B_{n_{l,k}}(0) - f(n_{l,k})T_{k-1} \\ & - \sum_{d=1}^{k-1} \sum_{\varphi_i \in \Phi_{l,k}(d)} (T_d - T_{d-1})\lambda(\varphi_i)a(\varphi_i, \varphi_{i+1})\epsilon_r L \\ & - \sum_{d=1}^{k-1} \sum_{\delta_i \in \Delta_{l,k}(d)} (T_d - T_{d-1})\lambda(\delta_i)b(\delta_i, \delta_{i+1})\epsilon_r L_a, \\ & \forall l = 2..h_k. \end{aligned} \tag{17}$$

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Lemma 1. *When discovered routes are minimum-hop routes and ARQ is supported,* $0 \leqslant \gamma_{l,k}(d) \leqslant 3$ *and* $0 \leqslant \gamma'_{l,k}(d) \leqslant 3$ $\forall l = 2..h_k$ *and* $\forall k, d < k$.

Proof. The proof follows from Theorem 1. □

To summarize, we can use the expressions provided in this section to determine the energy consumption rate of nodes and their remaining battery energy, when ARQ is supported. On the basis of the provided formulation, we can determine the communication lifetime between two specific nodes i and j when ARQ is supported by the MAC layer. To find this, we can use Algorithm 1. Nevertheless, when ARQ is supported, the related procedure explained in Algorithm 2 must be replaced by Algorithm 3.

Algorithm 3. Procedure Update $\mathbb{B}$ and $\mathbb{G}$ when ARQ is supported.

```
B_{n_{1,k}} ← B_{n_{1,k}} − C(n_{l,k})(T_k − T_{k−1})
B_{n_{h_k+1,k}} ← B_{n_{h_k+1,k}} − C(n_{h_k+1,k})(T_k − T_{k−1})
for u = 1 to N & u ∉ P_k do
  B_u ← B_u − f(u)(T_k − T_{k−1})
  for l = 1 to h_k do
    if (u, n_{l,k}) ∈ E then
      B_u ← B_u − λ(n_{l,k})a(u,n_{l,k}) ϵ_r L(T_k − T_{k−1})
    end if
  end for
  for l = 2 to h_k + 1 do
    if (u, n_{l,k}) ∈ E then
      B_u ← B_u − λ(n_{l,k})b (u,n_{l,k})ϵ_r L_a(T_k − T_{k−1})
    end if
  end for
  if B_u < 0 then
    Remove u from G
  end if
end for
Remove P_k − {n_{1,k}, n_{h_k+1,k}} from G
```

6. Bounds on the lifetime of node-disjoint routes

To derive upper and lower bounds on the lifetime of node-disjoint routes, we notice that lifetime of a route is defined as the time at which one of its relay nodes (not the source or the destination) fails. Therefore, to determine the route lifetime, we represent the expression given for T_k in (1) as follows:

$$T_k = T_{k-1} + D_k, \quad \forall k = 1, 2, \ldots \tag{18}$$

in which $D_k = \min\left(\frac{B_{n_{2,k}}(T_{k-1})}{c(n_{2,k})}, \ldots, \frac{B_{n_{h_k,k}}(T_{k-1})}{c(n_{h_k,k})}\right)$ is the duration that the kth route could be used for packet transfer from the source to the destination assuming the source and the destination do not fail. D_k (and hence T_k) may take different values for different pairs of source–destination nodes in the network, because the remaining battery energy and the energy consumption rate of relay nodes in the kth route between different pairs of source–destination nodes might be different. Here, we determine lower and upper bounds of T_k for any pair of source–destination nodes in the network. The following inequalities follow from the definition of D_k:

$$\frac{B_{low}(T_{k-1})}{c_{up}} \leqslant D_k \leqslant \frac{B_{up}(T_{k-1})}{c_{low}}, \quad \forall k = 1, 2, \ldots \tag{19}$$

in which, $B_{low}(T_{k-1})$ and $B_{up}(T_{k-1})$ are the lower and upper bounds for the remaining battery energy of a node in the k^{th} node-disjoint route between a pair of source–destination nodes at time T_{k-1}, respectively. Furthermore, c_{low} and c_{up} are the lower and upper bounds for the energy consumption rate of a relay node in a multi-hop route, respectively. If we replace (19) in (18), we have

$$T_{k-1} + \frac{B_{low}(T_{k-1})}{c_{up}} \leqslant T_k \leqslant T_{k-1} + \frac{B_{up}(T_{k-1})}{c_{low}}, \quad \forall k = 1, 2, \ldots \tag{20}$$

6.1. Networks without ARQ support

When ARQ is not supported, the energy consumption rate of relay nodes, $c(n_{l,k})$, was specified in (5). An upper bound for $c(n_{l,k})$ is achieved when a relay node receives and transmits packets with the highest rate, and overhears packets transmitted by its downstream relay node with the highest rate. The highest packet forwarding rate for a relay node is the rate at which the source transmits packets (i.e., λ). Thus, using (5), we can show that

$$c(n_{l,k}) < \begin{cases} \lambda L(2\epsilon_r + \epsilon_t) + f_{max}, & \forall l = 2..h_k - 1, \\ \lambda L(\epsilon_r + \epsilon_t) + f_{max}, & l = h_k, \end{cases} \tag{21}$$

in which, $f_{max} = \max\{f(u)\}_{u=1}^{N}$ is the largest idle-mode energy consumption among nodes of the network. From (21), we can conclude that the upper bound of $c(n_{l,k})$ is

$$c_{up} = \lambda L(2\epsilon_r + \epsilon_t) + f_{max}.$$

In theory, a lower bound for $c(n_{l,k})$ is achieved when a relay node receives packets with the lowest possible rate. According to Remark 2, the lowest rate that a relay node in a multi-hop route can receive packets belongs to the last relay in the route. However, in practice, the route lifetime will be dominated by the lifetime of a node which has the highest energy consumption rate in the route. According to (5), the lowest energy consumption rate for a relay node that its death can cause route failure belongs to the relay in a route with two hops. Therefore, a lower bound for $c(n_{l,k})$ is

$$c_{low} = \lambda L(\epsilon_r + p_{min}\epsilon_t) + f_{min},$$

in which $p_{min} = \min(p(u, v)),\ \forall(u, v) \in \mathbb{E}$, and $f_{min} = \min\{f(u)\}_{u=1}^{N}$ is the lowest idle-mode energy consumption among nodes of the network.

To determine lower and upper bounds of the remaining battery energy of a relay node in a route, $B_{low}(T_{k-1})$ and $B_{up}(T_{k-1})$, we use Corollary 1, which indicates a relay node can at most overhear three nodes from other routes. Hence, $B_{low}(T_{k-1})$ is achieved for $\gamma_{l,k}(d) = 3, \forall d = 1\ldots k-1$ in (7). Furthermore, all three overheard nodes in each route must transmit with the highest packet forwarding rate for a node in a multi-hop route- (i.e., λ). If so, $B_{low}(T_{k-1})$ is obtained as

$$B_{low}(T_{k-1}) = B - (f_{max} + 3\lambda\epsilon_r L)T_{k-1}.$$

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The upper bound, $B_{up}(T_{k-1})$, is achieved if no node from the other routes are overheard. In other words, we must consider $\gamma_{l,k}(d) = 0$, $\forall d = 1 \ldots k-1$, in (7). Hence, we will have,

$$B_{up}(T_{k-1}) = B - f_{min}T_{k-1}.$$

By replacing the expressions found for $B_{low}(T_{k-1})$, $B_{up}(T_{k-1})$, c_{low}, and c_{up}, in (20), we arrive at the following expression:

$$A_1 + (1-A_2)T_{k-1} \leqslant T_k \leqslant A_3 + (1-A_4)T_{k-1}, \quad (22)$$

where

$$\begin{cases} A_1 = \dfrac{B}{\lambda L(2\epsilon_r+\epsilon_t)+f_{max}}, & A_2 = \dfrac{f_{max}+3\lambda L\epsilon_r}{\lambda L(2\epsilon_r+\epsilon_t)+f_{max}}, \\ A_3 = \dfrac{B}{\lambda L(\epsilon_r+p_{min}\epsilon_t)+f_{min}}, & A_4 = \dfrac{f_{min}}{\lambda L(\epsilon_r+p_{min}\epsilon_t)+f_{min}}. \end{cases}$$

If we look at the definition of A_4, we realize that $0 < A_4 < 1$, which means $0 < 1 - A_4 < 1$. We can also show that if $\epsilon_t > \epsilon_r$, $A_2 < 1$ (hence, $0 < 1 - A_2 < 1$).

The inequalities in (22) are recursive. To resolve the recursive dependency in (22), we replace T_{k-1} in the left hand side of (22) by the lower bound of T_{k-1} and T_{k-1} in the right hand side of (22) by the upper bound of T_{k-1}. As a result, the recursive inequalities in (22) could be represented as follows:

$$A_1\left(1+(1-A_2)+\cdots+(1-A_2)^{k-1}\right) \leqslant T_k \leqslant A_3\left(1+(1-A_4)+\cdots+(1-A_4)^{k-1}\right). \quad (23)$$

We can simplify the geometric series in (23) to arrive at the upper bound and lower bound of T_k as follows:

$$\begin{cases} T_{k_{low}} = A_1\frac{1-(1-A_2)^k}{A_2}, & \forall k = 1,2,\ldots, \\ T_{k_{up}} = A_3\frac{1-(1-A_4)^k}{A_4}, & \forall k = 1,2,\ldots \end{cases}$$

The following theorem summarizes this section:

Theorem 2. *Let $f_{min} = \min\{f(u)\}_{u=1}^N$ and $f_{max} = \max\{f(u)\}_{u=1}^N$. If ARQ is not supported at the MAC layer, then $T_{k_{low}} \leqslant T_k \leqslant T_{k_{up}}$ in which $T_{k_{low}} = A_1\frac{1-(1-A_2)^k}{A_2}$ and $T_{k_{up}} = A_3\frac{1-(1-A_4)^k}{A_4}$ and A_1 to A_4 are defined as*

$$\begin{cases} A_1 = \dfrac{B}{\lambda L(2\epsilon_r+\epsilon_t)+f_{max}}, & A_2 = \dfrac{f_{max}+3\lambda L\epsilon_r}{\lambda L(2\epsilon_r+\epsilon_t)+f_{max}}, \\ A_3 = \dfrac{B}{\lambda L(\epsilon_r+p_{min}\epsilon_t)+f_{min}}, & A_4 = \dfrac{f_{min}}{\lambda L(\epsilon_r+p_{min}\epsilon_t)+f_{min}}. \end{cases}$$

6.2. Networks with ARQ support

When ARQ is supported, the energy consumption rate of relay nodes, $c(n_{l,k})$, was specified in (14) for routes with two hops, in (15) for routes with three hops, and in (16) for routes with more than three hops. An upper bound for $c(n_{l,k})$ is achieved when a relay node receives and transmits packets with the highest rate, and overhears both data packets and ACKs transmitted by its downstream and upstream relay nodes. This situation happens in a route with more than three hops. Thus, from (16), we have

$$c_{up} = \lambda(a_{max}L + b_{max}L_a)(\epsilon_t + 2\epsilon_r) + f_{max},$$

in which $a_{max} = \max(a(u,v))$, $\forall(u,v) \in \mathbb{E}$, and $b_{max} = \max(b(u,v))$, $\forall(u,v) \in \mathbb{E}$.

A lower bound for $c(n_{l,k})$ is achieved when a relay node receives packets with the lowest possible rate. According to Remark 2, the lowest rate that a relay node in a multihop route can receive packets belongs to the last relay in the route.[2] However, in practice, the route lifetime will be dominated by the lifetime of a node which has the highest energy consumption rate in the route. When ARQ is supported, the lowest energy consumption rate for a relay node that its death can cause route failure happens in a route with two hops, when each data packet and its ACK are transmitted only once. Thus, using (14), we have

$$c_{low} = \lambda(L + L_a)(\epsilon_r + \epsilon_t) + f_{min}.$$

To determine the lower bound of the remaining battery energy of a relay node in a route when ARQ is supported, $B_{low}(T_{k-1})$, we use Lemma 1, which indicates that a relay node can overhear data packets and ACKs at most from three nodes in the other routes. By replacing $\gamma_{l,k}(d) = 3$ and $\gamma'_{l,k}(d) = 3\ \forall d = 1 \ldots k-1$ in (17), $B_{low}(T_{k-1})$ is achieved as follows:

$$B_{low}(T_{k-1}) = B - (f_{max} + 3\lambda(a_{max}L + b_{max}L_a)\epsilon_r)T_{k-1}.$$

The upper bound, $B_{up}(T_{k-1})$ is achieved when no node from the other routes are overheard. By replacing $\gamma_{l,k}(d) = 0$ and $\gamma'_{l,k}(d) = 0\ \forall d = 1 \ldots k-1$ in (17), $B_{up}(T_{k-1})$ is as follows:

$$B_{up}(T_{k-1}) = B - f_{min}T_{k-1}.$$

By replacing the expressions found for $B_{low}(T_{k-1})$, $B_{up}(T_{k-1})$, c_{low}, and c_{up}, in (20), we arrive at the following inequalities:

$$H_1 + (1-H_2)T_{k-1} \leqslant T_k \leqslant H_3 + (1-H_4)T_{k-1}, \quad (24)$$

where

$$\begin{cases} H_1 = \dfrac{B}{\lambda(a_{max}L+b_{max}L_a)(\epsilon_t+2\epsilon_r)+f_{max}}, \\ H_2 = \dfrac{f_{max}+3\lambda(a_{max}L+b_{max}L_a)\epsilon_r}{\lambda(a_{max}L+b_{max}L_a)(\epsilon_t+2\epsilon_r)+f_{max}}, \\ H_3 = \dfrac{B}{\lambda(L+L_a)(\epsilon_r+\epsilon_t)+f_{min}}, \\ H_4 = \dfrac{f_{min}}{\lambda(L+L_a)(\epsilon_r+\epsilon_t)+f_{min}}. \end{cases}$$

If we look at the definition of H_4, we realize that $0 < H_4 < 1$, which means $0 < 1 - H_4 < 1$. Assuming $\epsilon_t > \epsilon_r$, we can also show that $H_2 < 1$ ($0 < 1 - H_2 < 1$). Since both $1 - H_2$ and $1 - H_4$ are positive values, we can replace T_{k-1} in the left hand side of (24) by the lower bound of T_{k-1} and T_{k-1} in the right hand side of (22) by the upper bound of T_{k-1} to widen the bounds. If so, we arrive at the following expression:

$$H_1\left(1+(1-H_2)+\cdots+(1-H_2)^{k-1}\right) \leqslant T_k \leqslant H_3\left(1+(1-H_4)+\cdots+(1-H_4)^{k-1}\right). \quad (25)$$

[2] This remark was mentioned for the case in which ARQ is not supported. However, it is true, even if ARQ is supported.

We can simplify the geometric series in (25) to arrive at the upper and lower bound of T_k, when ARQ is supported, as follows:

$$\begin{cases} T_{k_{low}} = H_1 \frac{1-(1-H_2)^k}{H_2}, & \forall k = 1,2,\ldots, \\ T_{k_{up}} = H_3 \frac{1-(1-H_4)^k}{H_4}, & \forall k = 1,2,\ldots \end{cases}$$

The following theorem summarizes this section:

Theorem 3. *Let $f_{min} = \min\{f(u)\}_{u=1}^N$ and $f_{max} = \max\{f(u)\}_{u=1}^N$. If ARQ is supported by the MAC layer, then $T_{k_{low}} \leqslant T_k \leqslant T_{k_{up}}$ in which $T_{k_{low}} = H_1 \frac{1-(1-H_2)^k}{H_2}$ and $T_{k_{up}} = H_3 \frac{1-(1-H_4)^k}{H_4}$ and H_1 to H_4 are defined as*

$$\begin{cases} H_1 = \dfrac{B}{\lambda(a_{max}L + b_{max}L_a)(\epsilon_t + 2\epsilon_r) + f_{max}}, \\ H_2 = \dfrac{f_{max} + 3\lambda(a_{max}L + b_{max}L_a)\epsilon_r}{\lambda(a_{max}L + b_{max}L_a)(\epsilon_t + 2\epsilon_r) + f_{max}}, \\ H_3 = \dfrac{B}{\lambda(L + L_a)(\epsilon_r + \epsilon_t) + f_{min}}, \\ H_4 = \dfrac{f_{min}}{\lambda(L + L_a)(\epsilon_r + \epsilon_t) + f_{min}}. \end{cases}$$

7. Expected node-to-node communication lifetime

In Sections 4 and 5, we determined maximum communication lifetime between two specific nodes in the network. In this section, we determine the expected value of this lifetime in ad hoc networks with a random topology.- To this aim, we obviously need to know the probability distribution function (PDF) of the node-to-node communication lifetime in the network. As we saw in Sections 4 and 5, node-to-node communication lifetime depends on the reliability of links. Therefore, its PDF is also dependent on the PDF of the reliability of links in the network. Reliability of links, in turn, depends on modulation and channel coding schemes deployed at the physical layer as well as the channel fading model (e.g., Rayleigh or Rician models). These dependencies make calculating of the expected node-to-node communication lifetime in ad hoc networks with random topology very complicated. Furthermore, any provided analysis will be dependent on the assumed modulation and channel coding schemes. Considering the effect of overhearing and assuming different initial battery energy and idle-mode energy consumption rate for different nodes add to the complexity of the problem.

We, however, provide an analysis to approximate the expected value of the maximum lifetime between any two nodes in the network under some assumptions. The importance of this approximation is that it gives a closed-form expression for the maximum duration that two nodes can communicate with each other in an ad hoc network with a random topology. In the next section, we will use simulation results to show that this approximate expression is of a good accuracy even if we relax some of the assumptions.

These are our assumptions:

(1) All nodes have the same idle-mode energy consumption rate, i.e., $f(u) = f,\ \forall u \in \mathbb{V}$.
(2) The packet delivery ratio of various links is 1 (perfect link), i.e., $p(u,v) = 1$ and $q(u,v) = 1,\ \forall(u,v) \in \mathbb{E}$.
(3) All nodes have the maximum battery energy at the network start up, i.e., $B_u(0) = B,\ \forall u \in \mathbb{V}$.

With these assumptions, we present the analysis for networks with ARQ support. The same analysis is valid for networks without ARQ only if we set $L_a = 0$ in the equations presented in this section.

To calculate the expected communication lifetime in the network, we again choose two nodes randomly. They could be either neighbor or several hops away from each other. Let $\overline{T}$ be the expected value of the communication lifetime of two nodes in the network. Furthermore, let $\overline{T}_{sh}$ be the expected communication lifetime if the two randomly chosen nodes are neighbor, and $\overline{T}_{mh}$ be the expected communication lifetime if they are not neighbor. The probability that the two nodes are neighbors is the probability that the destination node (which is chosen randomly) is within the transmission range of the source node. Since nodes are assumed to be uniformly distributed in the network, we have $\Pr\{\text{connection is single hop}\} = \pi d_{max}^2/A$, where d_{max} is the transmission range and A is the network area. Therefore, we have

$$\overline{T} = \frac{\pi d_{max}^2}{A}\overline{T}_{sh} + \left(1 - \frac{\pi d_{max}^2}{A}\right)\overline{T}_{mh}. \quad (26)$$

7.1. Expected communication lifetime of neighboring nodes

Since $p(u,v) = 1$ and $q(u,v) = 1$, we have $a(u,v) = 1$ and $b(u,v) = 1,\ \forall(u,v) \in \mathbb{E}$. Considering (11) and (8), the energy consumption rate of the source node and the destination node, when they are neighbors, are respectively as follows:

$$\begin{cases} c_{s_s} = \lambda(L\epsilon_t + L_a\epsilon_r) + f, \\ c_{d_s} = \lambda(L\epsilon_r + L_a\epsilon_t) + f. \end{cases}$$

Thus, the lifetime of the source and the destination node when they are neighbors are respectively as follows:

$$\begin{cases} T_{s_s} = \frac{B}{c_{s_s}} = \frac{B}{\lambda(L\epsilon_t + L_a\epsilon_r) + f}, \\ T_{d_s} = \frac{B}{c_{d_s}} = \frac{B}{\lambda(L\epsilon_r + L_a\epsilon_t) + f}. \end{cases}$$

Assuming $\epsilon_t > \epsilon_r$ and $L > L_a$, the communication lifetime of two neighboring nodes is-

$$T_{sh} = \min(T_{s_s}, T_{d_s}) = \frac{B}{\lambda(L\epsilon_t + L_a\epsilon_r) + f}. \quad (27)$$

Since there is no random variable accessioned with T_{sh}, we have

$$\overline{T}_{sh} = \frac{B}{\lambda(L\epsilon_t + L_a\epsilon_r) + f}. \quad (28)$$

7.2. Expected communication lifetime of non-neighboring nodes

To find the expected lifetime of communication between non-neighboring nodes, we first assume that the number of available node-disjoint routes between the source and the destination is known. Then, we take into account the effect of randomness of the number of available routes between two arbitrary nodes in the network.

Assuming $p(u,v) = 1$ and $q(u,v) = 1,\ \forall (u,v) \in \mathbb{E}$ and considering (13), we can express the energy consumption rate of the source node and the destination node as-

$$\begin{cases} c_{s_m} = \lambda(L\epsilon_t + L_a\epsilon_r) + \lambda\epsilon_r L + f, \\ c_{d_m} = \lambda(L\epsilon_r + L_a\epsilon_t) + \lambda\epsilon_r L_a + f. \end{cases} \tag{29}$$

The first term in the expression given for c_{s_m} in (29) is the energy consumed by the source node to transmit λ packets per second, and the second term is the energy consumed during overhearing of the packets forwarded by the first relay node. Similarly, the first term in c_{d_m} is the energy consumed by the destination to receive a packet, and the second term is the energy consumed to overhear ACKs sent by the last relay. Therefore, when the source and the destination are not neighbors, their lifetimes will be respectively as follows:

$$\begin{cases} T_{s_m} = \frac{B}{\lambda(L\epsilon_t + (L_a + L)\epsilon_r) + f}, \\ T_{d_m} = \frac{B}{\lambda((L + L_a)\epsilon_r + L_a\epsilon_t) + f}. \end{cases} \tag{30}$$

We recall that the energy consumption rate of relay nodes depends on the hop-count of the route that they are part of. Nevertheless, if we assume the probability that two arbitrary nodes in the network are more than three hops away from each other (more than two hops, in case of no ARQ) is much higher than the probability that they are less than three hops away from each other, using (16) we can calculate the energy consumption rate of a relay node as follows:

$$c_r = \lambda((L + L_a)(2\epsilon_r + \epsilon_t)) + f. \tag{31}$$

Now, we can compute the lifetime of the first route as

$$\overline{T}_1 = \frac{B}{\overline{c}_r}. \tag{32}$$

To compute the lifetime of other node-disjoint routes, we need to determine the amount of energy remaining for a relay node in these routes at the time that the previously in-use route fails. For this, we define $\bar{\gamma}_k$ as the expected number of nodes in P_k which have physical links with a node from the previous routes P_d, $\forall d = 1..k-1$. In the worst case, a relay node whose death causes route failure, overhears both ACKs and data packets transmitted by a node in the in-use route. Therefore, using (17), we can calculate the amount of battery energy remaining for such a relay node in P_k as

$$B_k = B - T_{k-1}[\bar{\gamma}_k\lambda\epsilon_r(L + L_a) + f],\ \forall k = 1,2,..K,$$

where K is the number of node-disjoint routes between the source and the destination node. The expected lifetime of the kth route between a source and a destination node in the network will be as follows:

$$T_k = T_{k-1} + \frac{B_k}{c_r}, \quad \forall k = 1,2,..K, \tag{33}$$

in which $T_0 = 0$. The recursive equation in (33) could be simplified to

$$T_k = \frac{B}{c_r}\sum_{i=0}^{k-1}\rho^i, \quad \forall k = 1,2,..K. \tag{34}$$

where

$$\rho = 1 - \frac{\bar{\gamma}_k\lambda\epsilon_r(L + L_a) + f}{c_r}. \tag{35}$$

From Theorem 1, we have $\max(\bar{\gamma}_k) = 3$. Considering this fact, we can show that if $\epsilon_t > \epsilon_r$, then $\rho < 1$. Therefore, the geometric series in (34) could be further simplified. We can arrive at the following closed-form expression for the expected lifetime of the kth node-disjoint route between a source and a destination node:

$$T_k = \left(\frac{B}{c_r}\right)\left(\frac{1 - \rho^k}{1 - \rho}\right), \quad \forall k = 1,2,\ldots K. \tag{36}$$

Knowing T_k, T_{s_m}, and T_{d_m}, we can calculate the communication lifetime of non-neighboring nodes as follows:

$$T_{mh} = \min(T_{s_m}, T_K, T_{d_m}), \tag{37}$$

in which T_K is the lifetime of the last available route between a source and a destination node, which is obtained using (36) for $k = K$.

Assuming $\epsilon_t > \epsilon_r$, we can show that

$$\min(T_{s_m}, T_{d_m}) = T_{s_m} = \frac{B}{\lambda(L\epsilon_t + (L_a + L)\epsilon_r) + f}.$$

Hence, from (37) we conclude that

$$T_{mh} = \begin{cases} T_K, & \text{if } \frac{1-\rho^K}{1-\rho} < \frac{c_r}{c_{s_m}}, \\ T_{s_m}, & \text{if } \frac{1-\rho^K}{1-\rho} > \frac{c_r}{c_{s_m}}. \end{cases} \tag{38}$$

The fact that whether $\frac{1-\rho^K}{1-\rho} < \frac{c_r}{c_{s_m}}$ or $\frac{1-\rho^K}{1-\rho} > \frac{c_r}{c_{s_m}}$ depends on the value of K; the number of node-disjoint routes between the source and the destination nodes. For an arbitrary pair of source–destination nodes K could be a random variable, because nodes are distributed uniformly. Therefore, depending on the value of K, T_{mh} may take a value from the set $\{0, T_{s_m}, T_1, T_2, T_3, T_4, \ldots\}$. To determine the probability that T_{mh} takes one of these values, we first need to determine the minimum value of K which meets the inequality of $\frac{1-\rho^K}{1-\rho} > \frac{c_r}{c_{s_m}}$. Let this minimum value of K be denoted by K^*. Since $\frac{1-\rho^K}{1-\rho}$ is the summation of a geometric series with $K-1$ positive elements, if $\frac{1-\rho^K}{1-\rho} > \frac{c_r}{c_{s_m}}$ is true for $K = K^*$, it will be true for $K > K^*$ as well. Hence, we can say that

$$T_{mh} = \begin{cases} T_{s_m}, & \text{with } \Pr\{K \geqslant K^*\}, \\ T_k, & \text{with } \Pr\{K = k\},\ \forall k = 0\ldots K^* - 1. \end{cases} \tag{39}$$

In (39), $\Pr\{K \geqslant K^*\}$ is the probability that there are at least K^* node-disjoint routes between the source and the destination node, while $\Pr\{K = k\}$ is the probability that there are exactly k node-disjoint routes between them. If we define

$$\sigma(k) = \Pr\{K \geqslant k\}, \tag{40}$$

then $\Pr\{K = k\}$ is computed as,

$$\Pr\{K = k\} = \sigma(k) - \sigma(k+1). \tag{41}$$

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Therefore, the expected value of T_{mh} is obtained as follows:

$$\overline{T}_{mh} = T_{s_m}\sigma(K^*) + \sum_{k=0}^{K^*-1} T_k[\sigma(k) - \sigma(k+1)]. \quad (42)$$

To find an expression for $\sigma(k)$, we define $\theta(k)$ as the probability that there are at least k node-disjoint routes between *every* two nodes in the network. Note that $\sigma(k)$ is the probability that there are at least k node-disjoint routes between two *arbitrary* nodes. The former event is stricter than the latter event. Thus,

$$\sigma(k) \geqslant \theta(k). \quad (43)$$

Considering (42) and the inequality in (43), we have shown in Appendix that

$$\overline{T}_{mh} \geqslant T_{s_m}\theta(K^*) + \sum_{k=0}^{K^*-1} T_k[\theta(k) - \theta(k+1)]. \quad (44)$$

Since $\epsilon_t > \epsilon_r$, if we replace $\overline{T}_{mh}$ from (44) and $\overline{T}_{sh}$ from (28) in (26), we arrive at the following expression for the expected communication lifetime of two nodes in the network:

$$\overline{T} \geqslant \frac{\pi d_{max}^2}{A} T_{s_s} + \left(1 - \frac{\pi d_{max}^2}{A}\right)\left[T_{s_m}\theta(K^*) + \sum_{k=1}^{K^*-1} T_k[\theta(k) - \theta(k+1)]\right]. \quad (45)$$

Eq. (45) is our closed-form expression for the expected node-to-node communication lifetime in wireless ad hoc networks with random topology. Although, (45) gives a lower bound, we show in the next section using simulation results that it is a very tight bound and could be considered as an approximate expression for the expected communication lifetime of two nodes.

Note that in (45), we can approximate $\theta(k)$ using the theory of connectivity of wireless ad hoc networks. An ad hoc network is called k-connected (k-vertex-connected), if there are at least k node-disjoint routes between *every* two nodes in that network. If nodes in an ad hoc network are uniformly distributed in an square area, $\theta(k)$ is approximated as follows [1]:

$$\theta(k) = \left(1 - e^{-\tau}\sum_{i=0}^{k-1}\frac{\tau^i}{i!}\right)^N,$$

where $\tau = N\pi d_{max}^2/A$. Here, N is the number of nodes, d_{max} is the transmission range, and A is the area of the square field. The above approximation is subject to $\pi d_{max}^2/A \ll 1$ (i.e., the transmission area of a node must be much smaller than the network area).

8. Simulation results

In this section, we present simulation results to verify our analysis. To this aim, we first derive the energy consumption parameters, ϵ_t and ϵ_r, of some commercial wireless products to have an idea about typical values of these two parameters.

8.1. Energy consumption characteristics of the wireless interface

We recall that ϵ_t and ϵ_r are the amount of energy consumed by a transmitting and a receiving node to send and receive a single bit of a packet, respectively. Suppose that the current consumption of the interface during signal transmission and reception is $\mathcal{I}_t$ (A) and $\mathcal{I}_r$ (A), respectively. If the data rate of the interface is r bit/s, then we can calculate ϵ_t and ϵ_r as follows:

$$\begin{cases} \epsilon_t = \frac{\mathcal{V}\mathcal{I}_t}{r} \ \text{(J/bit)}, \\ \epsilon_r = \frac{\mathcal{V}\mathcal{I}_r}{r} \ \text{(J/bit)}, \end{cases}$$

in which $\mathcal{V}$ (V) is the supply voltage of the interface. Table 1 shows the nominal values of $\mathcal{V}$, $\mathcal{I}_t$, $\mathcal{I}_r$, and the resulting values of ϵ_t and ϵ_r for two IEEE 802.11b products and one IEEE 802.15.4 product. As we see, ϵ_t is greater than ϵ_r for the 802.11b products. For the 802.15.4 product, ϵ_t is slightly smaller than ϵ_r. Since the energy consumed for packet transmission and reception depends on the packet length, we have also shown the size of data packets and ACKs in Table 2 for 802.11b and 802.15.4 standards. The table shows that the size of data packets is at least two times greater than the size of ACKs in 802.11b standard, and six times greater in 802.15.4 standard.

8.2. Simulation set-up

Nodes are distributed uniformly in a square area of size $8d_{max} \times 8d_{max}$, where d_{max} is the transmission range of nodes. Different nodes are able to send and transmit data in parallel. The wireless link between them is modeled as a 2 Mbps link, and the medium access mechanism is CSMA/CA (in accordance to IEEE 802.11b). In our simulation model, quality of different links could be different from each other. To this aim, the probability of error-free reception of data packets over a wireless link is chosen randomly from the interval $[p_{d_{min}}, 1]$. When ARQ is supported, the probability of error-free reception of an ACK is computed accordingly considering the ratio between the length of ACKs and data packets. We also assume that idle-mode energy consumption rate of different nodes could be different from each other. To this end, idle-mode energy consumption rate of each node, $f(u)$, is chosen randomly from the interval $[f_{min}f_{max}]$ (J/s). Note that in practice packet delivery ratio of links and idle-mode energy consumption rate of nodes may not have uniform distribution. Here, we chose them randomly from a given interval only to ensure that different links have different qualities and different nodes have different idle-mode energy consumption rates.

Upon transmission (reception) of a packet of length L bits by a node, $\epsilon_t L$ ($\epsilon_r L$) J is deducted from its remaining battery energy. When the remaining battery energy of a node falls bellow a threshold B_{th}, the node is considered to be dead. When a node fails, we remove the failed links and update the network topology. To measure the communication lifetime of two specific nodes, we transmit packets from the source node to the destination node until the source or the destination node fails, or the last available route between them fails. *Nevertheless, instead of measuring*

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Table 1
Energy consumption parameters of three wireless products.

Standard	Product	$\mathcal{V}$ (V)	$\mathcal{I}_t$ (mA)	$\mathcal{I}_r$ (mA)	Data rate	Tx. range (m)	ϵ_t (nJ/bit)	ϵ_r (nJ/bit)
802.11b	Linksys WPC11	3.3	430	140	11 Mb/s	150	129	42
802.11b	Cisco CardBus	3.3	530	282	11 Mb/s	50	159	85
802.15.4	Sentilla tmote sky	3.3	19.5	21.8	250 Kb/s	125	257	287

Table 2
Size of data packets and MAC ACKs in 802.11b and 802.15.4 standards. All Values are per bit. Here, l is the size of the message received from the application layer.

Standard	Packet type	PHY + MAC header	IPv4 + UDP header	Total packet size
802.11b	Data	128 + 224	160 + 64	$L = 576 + l$
802.11b	MAC ACK	128 + 112	No upper layer header	$L_a = 240$
802.15.4	Data	24 + 168	160 + 64	$L = 416 + l$
802.15.4	MAC ACK	24 + 40	No upper layer header	$L_a = 64$

Table 3
Default values of simulation parameters.

Parameter	Value
Initial battery energy of each node (B)	10 J
Source packet transmission rate (λ)	1 packet/s
Energy consumed for transmission of a single bit (ϵ_t)	159 nJ (Cisco Interface)
Energy consumed for reception of a single bit (ϵ_r)	85 nJ (Cisco Interface)
Data packet size (L)	1088 bits
ACK packet size (L_a)	240 bits
Transmission range (d_{max})	70 m
Battery death threshold (B_{th})	0
Maximum transmission tries of ARQ mechanism (M)	7
Minimum delivery probability of data packets ($p_{d_{min}}$)	0.6
Minimum energy consumption rate f_{min}	$0.1\epsilon_t L$
Maximum energy consumption rate f_{max}	$2\epsilon_t L$

the time duration, we measure the communication lifetime in terms of *<u>the total number of packets</u>* *transmitted by the source node before the communication between the source and the destination fails.*

Table 3 shows the default value of various parameters that we use in our simulations. Considering $\epsilon_t = 160$ nJ/bit and $\epsilon_r = 80$ nJ/bit in the table means that 0.174 mJ is consumed for transmission of a packet of size $L = 1088$ bits and 0.087 mJ is consumed for reception of the packet. Since we assumed $B = 10$, each node has enough energy to transmit 57471 packets or receive 114942 packets of length 1088 bits. Values of f_{min} and f_{max} in the table are chosen in such a way that the idle-mode energy consumption rate of each node is in the same order of the consumed energy for transmission of a data packet (i.e., $\epsilon_t L$). With proliferation of energy-efficient networking protocols and low power devices (e.g., in wireless sensor networks), idle-mode energy consumption of nodes could be a small value in practice.

8.3. Estimating node-to-node communication lifetime using numerical algorithms

In this experiment, we generate a network randomly in each simulation run. Then, we choose randomly a pair of nodes as source and destination. The communication lifetime of these two nodes is determined using simulation. We also determine their communication lifetime using Algorithm 1 and its related procedures for networks with and without ARQ support (i.e., Algorithms 2 and 3, respectively). This procedure is repeated 1000 times to compute the average communication lifetime between two nodes in the network with the confidence level 98%. The experiment is conducted separately for networks with and without ARQ support.

Effect of the number of nodes: Fig. 9 shows the average node-to-node communication lifetime in terms of the total number of nodes in the network. The figure shows that the there is a good match between the simulation results and the results predicted by Algorithm 1 in both types of networks. We also observe that communication lifetime of nodes increases as the number of nodes in the network increases. The reason lies in the fact that when the number of nodes in the network increases, probability of communication failure between two nodes due to lack of routes decreases. In such a case, the communication lifetime of two nodes can reach its maximum value, which is the time that one of the two nodes fails. Comparison of Fig. 9(a) with Fig. 9(b) reveals that the average node-to-node communication lifetime in networks with ARQ support is smaller than that of networks without ARQ support. This is due to the fact that when ARQ is supported energy consumption rate of nodes increases due to retransmissions of data and ACK packets.

Here, we should emphasize that the good match between the average nod-to-node communication lifetime in simulations and theoretical results is not because the simulation model is simply based on the theoretical model. Attention should be paid to the relatively large confidence intervals in Fig. 9. This suggests that there are quite big deviations from the theoretical values in some simulation runs. The main reason behind these deviations is the randomness in network topology, pair of source–destination nodes, quality of links, and idle-mode energy consumption of nodes.

Effect of idle-mode energy consumption rate of nodes: In this experiment, we set f_{min} to $0.1\epsilon_t L$ and we increase the value of f_{max}. By increasing f_{max}, the idle-mode

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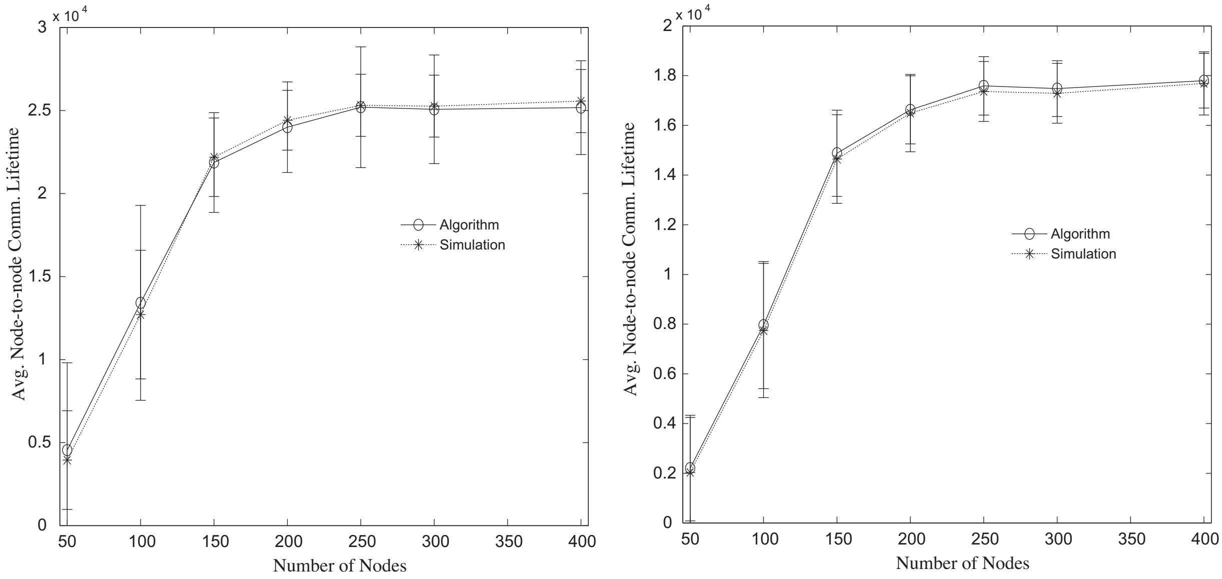
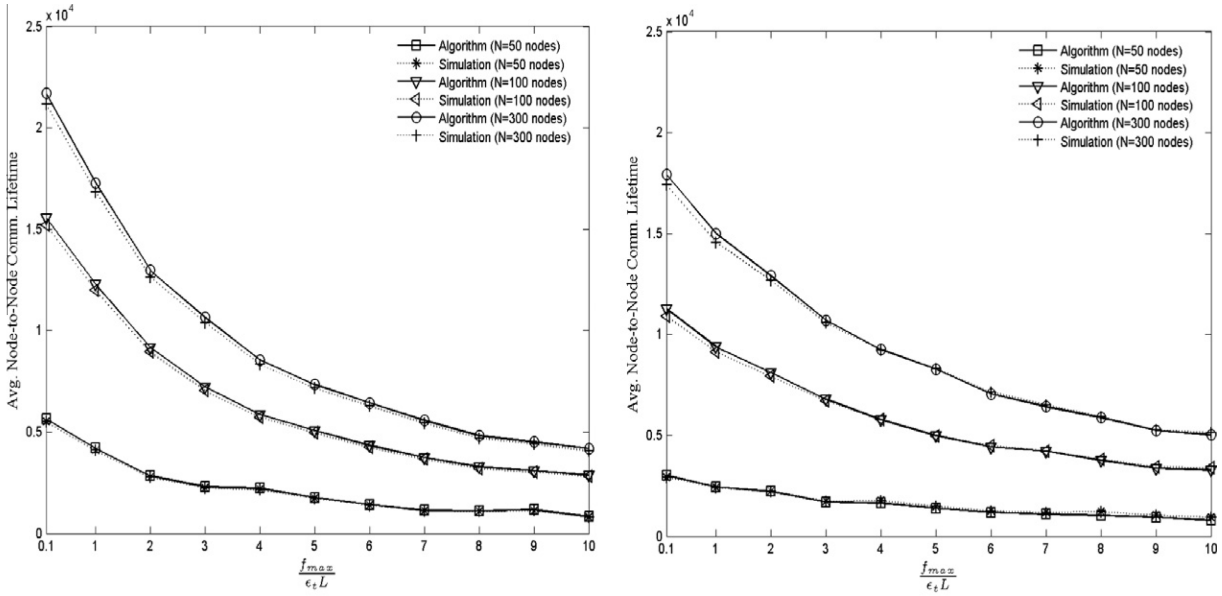


(a)WithoutARQ (b)WithARQ

Fig. 9. Average node-to-node communication lifetime as predicted by Algorithm 1 and simulations for different number of nodes in the network.



(a) Without ARQ (b) With ARQ

Fig. 10. Average node-to-node communication lifetime as predicted by Algorithm 1 and the simulation model for different value of f_{max}.

energy consumption rate of nodes increases on an average. We change f_{max} from $0.1\epsilon_t L$ to $10\epsilon_t L$. $f_{max} = 0.1\epsilon_t L$ represents a situation in which the idle-mode energy consumption rate of nodes is much smaller than the energy consumed for transmission of a data packet. On the other hand, $f_{max} = 10\epsilon_t L$ represents a situation in which the idle-mode energy consumption rate of a node could be several times greater than the energy consumed for transmission of a data packet. This may happen, for instance, when many control packets are overheard by nodes or their energy is highly consumed for non-communication purposes (e.g., in a laptop).

Fig. 10 shows the average node-to-node communication lifetime for networks with and without ARQ support as a function of $\frac{f_{max}}{\epsilon_t L}$. As we expect, when $\frac{f_{max}}{\epsilon_t L}$ increases, the average node-to-node communication lifetime decreases, because the energy consumption rate of nodes increases on average. When there are 300 nodes in the network and ARQ is not supported, the average node-to-node communication lifetime drops from 21701 to 4178, if f_{max} changes from $0.1\epsilon_t L$ to $10\epsilon_t L$ (i.e., 80% decrease). In the network with ARQ, it drops by around 70%. When there are 100 (50) nodes, it drops 81% (85%) in networks without ARQ and 68% (67%) in networks with ARQ. This highlights the importance of minimizing the idle-mode energy consumption rate of nodes compared to the energy that they consume for data communication in order to maximize the node-to-node communication lifetime.

8.4. Closed-form expression of expected node-to-node communication lifetime

In this section, we present results verifying the accuracy of the closed-form expression derived for expected communication lifetime of nodes in Section 7. In each simulation run, we generate a network randomly, and choose two nodes randomly. Then we measure their communication lifetime using the simulation model. The expected value of communication lifetime of nodes is computed by averaging over 1000 simulation runs. We also calculate the expected communication lifetime of nodes using the expression given in (45).

Accuracy of analytical results in the ideal case: We recall that the closed form expression in (45) derived under the assumption that $f(u) = f,\ \forall u \in \mathbb{V}$, and packet delivery ratio (PDR) of each link is 1 (i.e., $p(u, v) = 1\ \forall (u, v) \in \mathbb{E}$). Here, we present results for this ideal case. Then, we study the accuracy of the derived expression when we deviate from this ideal case. That is, when $f(u)$ is not the same for all nodes and PDRs of various links are different.

Fig. 11 shows the simulation and analytical results in terms of the number of nodes. Here, we assumed that $f(u) = \epsilon_t L_d$. Results have been shown for networks with and without ARQ separately. We observe a good match between simulation and analytical results in both types of networks. Analytical and simulation results follow the same trend when the number of nodes increases in the network. Remember that the expression given in (45) is in fact a lower bound for expected value of maximum node-to-node communication lifetime in the network. Nevertheless, since the lower bound is relatively tight as shown in Fig. 11, it could even be considered as an approximation for the expected value of maximum node-to-node communication lifetime in the network. That is,

$$\overline{T} \approx \frac{\pi d_{max}^2}{A} T_{s_s} + \left(1 - \frac{\pi d_{max}^2}{A}\right)\left[T_{s_m}\theta(K^*) + \sum_{k=1}^{K^*-1} T_k[\theta(k) - \theta(k+1)]\right]. \quad (46)$$

To study the accuracy of the expression given in (46) when we deviate from the ideal case, we first assume $f(u)$ for all nodes is the same, but $p(u,v)$ can take different values for different links. Then, we assume $p(u,v)$ is the same for all links, but $f(u)$ can take different values for different nodes. Finally, we assume both $f(u)$ and $p(u,v)$ can take different values for different nodes and links independent from each other.

Effect of PDR of links: Here, we set $f(u) = L\epsilon_t$, and we choose the value of $p(u,v)$ for each link from the interval $[p_{d_{min}}, 1]$. By decreasing the value of $p_{d_{min}}$, PDRs of links decrease on the average. Furthermore, PDRs vary more from one link to another link. That is, when $p_{d_{min}}$ decreases, we deviate more from the ideal case of having the perfect quality for all links.

When $p_{d_{min}}$ decreases, we observe different trends for node-to-node communication lifetime in networks with and without ARQ. Fig. 12(a) shows that in networks without ARQ support, the analytical results become less

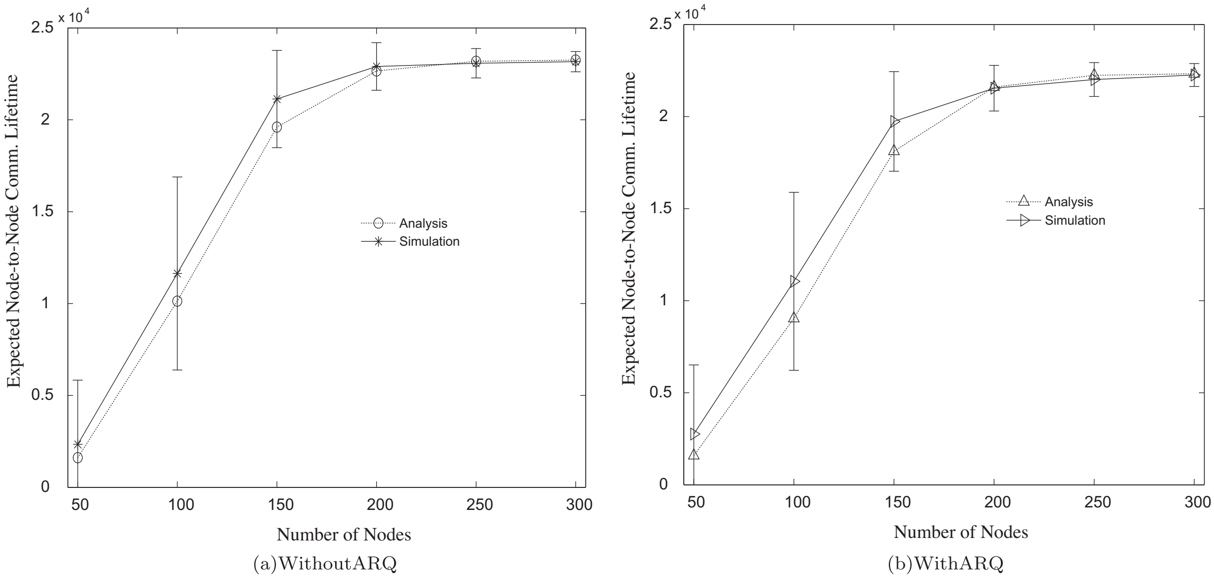


Fig. 11. Analytical and simulation results for the expected node-to-node communication lifetime in terms of the total number of nodes in the network. Idle-mode energy consumption rate is the same for different nodes and packet delivery ratio is one for all links.

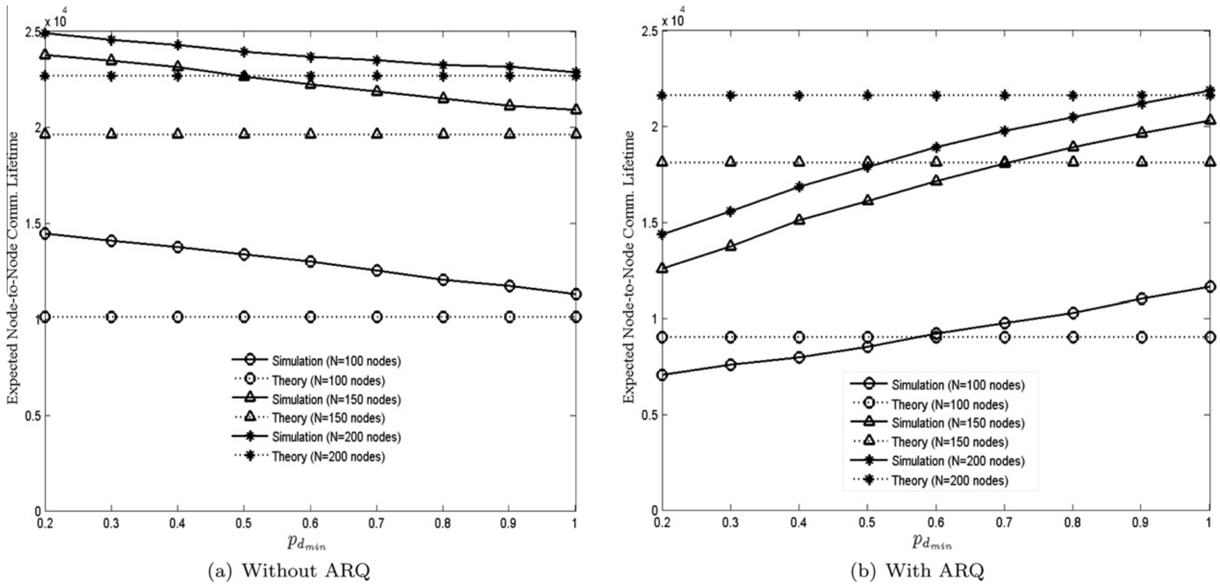


Fig. 12. The impact of quality of links on the analytical and simulation values of expected node-to-node communication lifetime in the network.

accurate as $p_{d_{min}}$ decreases. This is true regardless of the number of nodes in the network.

From plots of Fig. 12(a), we can also conclude that when ARQ is not supported, the node-to-node communication lifetime increases as quality of links decreases. The reason lies in the fact that nodes consume less amount of energy for packet forwarding, because they receive less number of packets which they have to forward. Of course, this increased lifetime is with the cost of having less number of packets delivered to their destinations.

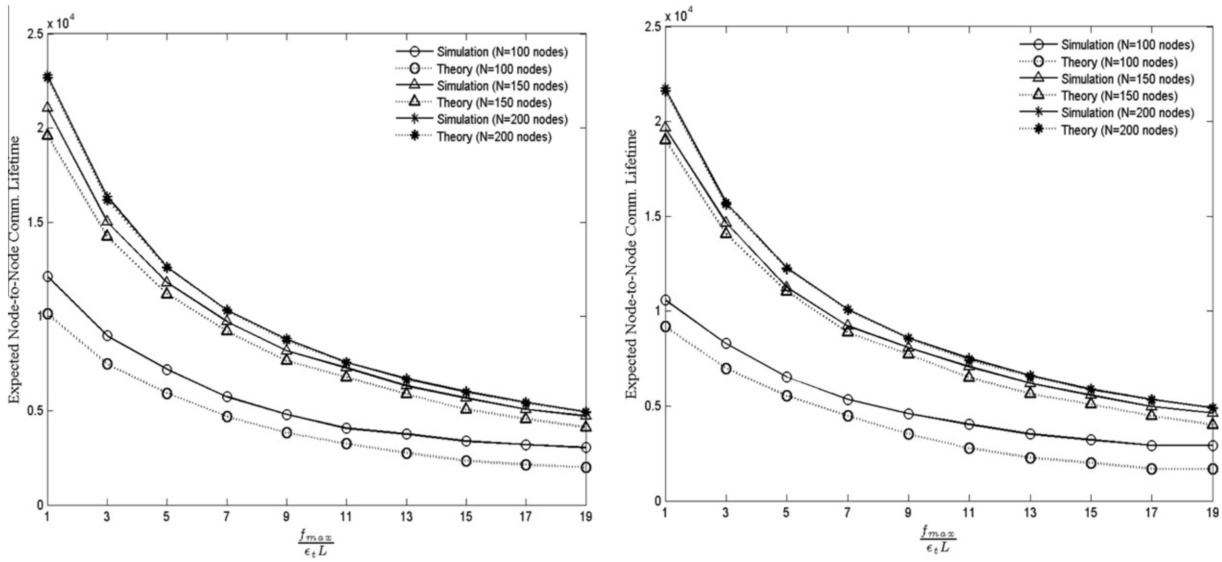
On the other hand, Fig. 12(b) shows that in network with ARQ support, the node-to-node communication lifetime decreases as quality of links decreases. When ARQ is supported, nodes have to consume more amount of energy to forward packets on low quality links, because they have to retransmit the same packet more times. When ARQ is supported, accuracy of the analytical results might even become better if $p_{d_{min}}$ decreases. Nevertheless, this depends on the number of nodes in the network. The most accurate results belong to the case of $p_{d_{min}} = 0.6$ for $N = 100$ nodes, $p_{d_{min}} = 0.7$ for $N = 150$ nodes, and $p_{d_{min}} = 1$ for $N = 200$ nodes.

Effect of idle-mode energy consumption rate of nodes: Here, we set $p(u,v) = 1$ for all links and we choose $f(u)$ for each node randomly from the interval $[f_{min}, f_{max}]$. We fix f_{min} to $\epsilon_t L_d$ and change f_{max} from $\epsilon_t L_d$ to $19\epsilon_t L_d$. We recall that (46) was derived assuming $f(u)$ is the same for all nodes. However, in this experiment $f(u)$ is different for different nodes. Here, the question is, if we want to use (46) when $f(u)$ is not same for all the nodes, what should be the value of f in the expressions given for c_{s_s}, c_{d_s}, c_{s_m}, c_{d_m}, and c_r in Section 7?.

As we mentioned before, we choose $f(u)$ for each node randomly between f_{min} and f_{max} for the sake of simulation. In practice, $f(u)$ may not have uniform distribution in the network. Although we might be able to derive another expression for expected node-to-node communication lifetime in the network assuming uniform distribution for $f(u)$, it may not be useful in practice. On the other hand, any analysis will be dependent on the distribution of $f(u)$ in the network, which itself depends on many factors such as wireless technology used, node density, and transmission range.

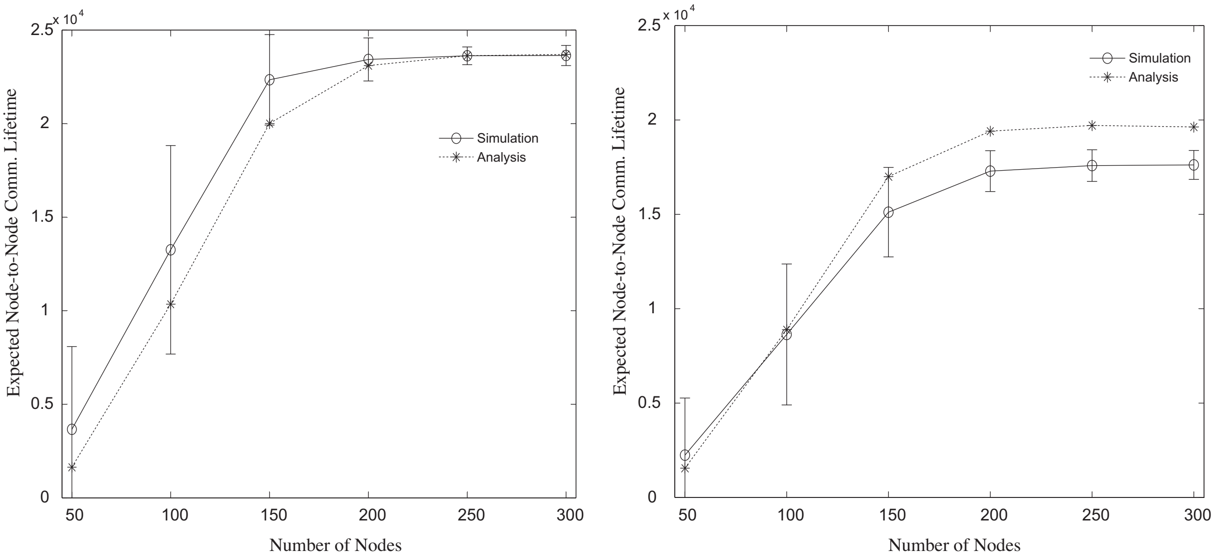
However, here we conjecture that if we replace f with the average idle-mode energy consumption rate of all nodes in the network, we still might be able to use (46) as an approximation for expected node-to-node communication lifetime in the network. That is, we assume $f = \frac{1}{N}\sum_{\forall u \in \mathbb{V}} f(u)$. With this assumption, we have plotted simulation and analytical values of expected node-to-node communication lifetime in Fig. 13 in terms of f_{max}. Fig. 13(a) and (b) show that even if the value of $f(u)$ varies from one node to another node, (46) is still accurate enough. The accuracy of the analytical results increase as the number of nodes increases in the network.

Joint effect of PDR of links and idle-mode energy consumption rate of nodes: Here, we choose $p(u,v)$ for each link randomly from the interval [0.6,1], and $f(u)$ for each node randomly from the interval $[0.1\epsilon_t L_d, 2\epsilon_t L_d]$. Fig. 14(a) and (b) show the simulation and analytical results in terms of the number of nodes in the network. The figures show that even if we deviate from the ideal case that (46) was derived for it, this expression is able to follow the increasing trend of expected node-to-node communication lifetime as the number of nodes increase. More specifically, when ARQ is not supported, analytical results are of high accuracy at higher number of nodes and acceptable accuracy at lower number of nodes. When ARQ is supported, analytical results are of high accuracy at lower number of nodes and acceptable accuracy at higher number of nodes.



(a) Without ARQ (b) With ARQ

Fig. 13. The impact of background traffic on analytical and simulation values of expected node-to-node communication lifetime in the network.



(a)WithoutARQ (b)WithARQ

Fig. 14. Analytical and simulations results for the expected node-to-node communication lifetime in the network. Idle-mode energy consumption rate is different for different nodes and packet delivery ratio is different for different links.

8.5. Lifetime of node-to-node communication for concurrent connections

So far, we studied node-to-node communication lifetime when there is only one active connection in the network. Nevertheless, in practice there might be concurrent communication in the network. In this section, we study node-to-node communication lifetime when there are more than one active connection in the network.

To this aim, at the network start up, we establish several connections between different pairs of source–destination nodes. The source–destination pairs are chosen randomly. Packets are transmitted between each pair of source–destination nodes until their connection fails due to battery exhaustion of the source or the destination node or due to lack of alternative routes. We repeated this experiment for $n \in \{1,2,3,4,8,12,20,40,80\}$ concurrent connections and for $N \in \{50,100,150,200,250\}$ nodes in the network. For each pair of values (n,N), the average value of the lifetime of all the established connections during 700 simulation runs is recorded as the average node-to-node communication lifetime.

Results in Fig. 15 show that when the number of connections increases, the average node-to-node communica-

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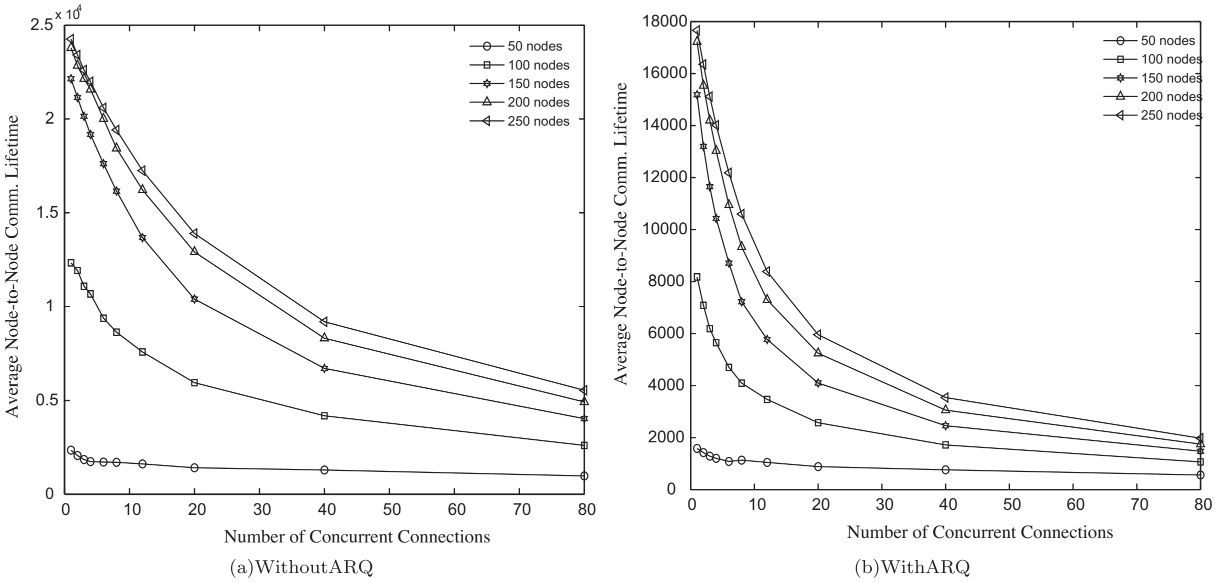

Fig. 15. Average node-to-node communication lifetime in networks with and without ARQ support when there are concurrent connections in the network.

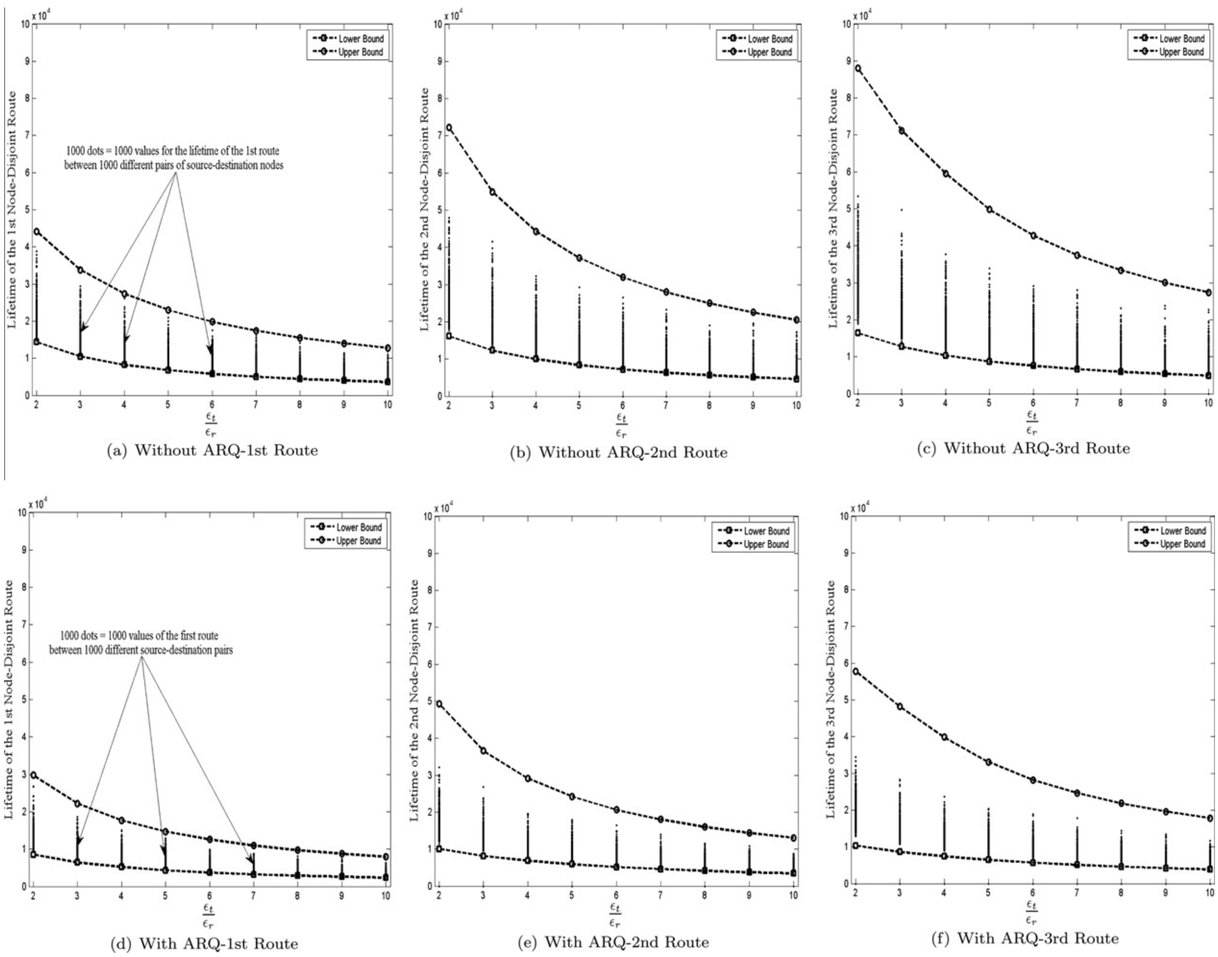

Fig. 16. Upper bound and the lower bound of the lifetime of the first, the second, and the third node-disjoint routes between a pair of source–destination nodes. Each dot between the two lines is a value obtained for one pair of source–destination nodes. In this experiment, $f(u)$ for each node has been chosen randomly from the interval $[0.1\epsilon_t L_d, 2\epsilon_t L_d]$.

tion lifetime converges to a constant value for each given number of nodes in the network. These constant values for different values of N seem to be close to each other. The rationale behind the convergence to a fixed value is that there always might be a possibility of a source and a destination node exchanging a small number of packets, especially when they are neighbors.[3] Only if the network is heavily dense and there are many concurrent communications, even two neighbors may not be able to exchange a packet due to very high probability of collision. This will cause nodes to die without successful packet exchange. In such cases, we may expect that the node-to-node communication lifetime to go to zero asymptotically.

Fig. 15 also shows that at higher number of nodes node-to-node communication lifetime decreases fast. Here, we explain the reason behind this behavior. Since the network area is fixed in this experiment, average node degree increases if the number of nodes increases. This, in turn, improves network connectivity [1]. If the number of connections increases in a network with higher degree of connectivity, energy consumption rate of nodes increases more. The reason lies in the fact that intermediate nodes in node-disjoint routes between a pair of source–destination node probably overhear more packets belonging to different connections. On the other hand, at lower densities, the probability that a node overhears packets belonging to different connections decreases. This also explains why the slope of the average communication lifetime is lower at lower densities.

8.6. Bounds on the lifetime of node-disjoint routes

In this section, we verify the accuracy of the upper and lower bounds of the lifetime of node-disjoint routes. We set the number of nodes in the network to 500 at which we observed that the network is likely to be 3-connected. We choose two nodes randomly in a randomly generated network. The time at which the first route between the selected nodes fails is recorded as the lifetime of the first route. After failure of the first route, the second route is used which is disjoint from the first route. Similarly, the time at which the second route fails is recorded as its lifetime and so on for the third route. To guarantee that the source and the destination nodes do not fail before routes, we set their initial battery energy to infinity. This procedure is repeated for 1000 source–destination pairs of nodes to measure the lifetime of the first, the second, and the third node-disjoint route between them.

We also computed the lower and the upper bounds using the expressions derived for them in Section 6. Results are shown in Fig. 16. The obtained value for each pair of nodes is shown by a dot in each figure (1000 dots for each point of the horizontal axis). We have plotted the results in terms of the ratio between the energy consumed by the wireless interface to transmit a single bit to the energy consumed to receive a single bit $\left(\frac{\epsilon_t}{\epsilon_r}\right)$.

Plots in Fig. 16 show that the derived bounds can accurately bind various values obtained for the lifetime of the first node-disjoint route between different pairs of source–destination nodes in both types of network (with and without ARQ support). For the second and third routes, the lower bound is still accurate in both types of networks. The upper bound, however, is not very accurate when $\frac{\epsilon_t}{\epsilon_r}$ is low. Nevertheless, as $\frac{\epsilon_t}{\epsilon_r}$ increases, the upper bounds become accurate as well.

9. Conclusion

It is important to find the duration for which two nodes in ad hoc networks can communicate with each other (referred to as node-to-node communication lifetime). In this paper, we analyzed the maximum lifetime of node-to-node communication in a static ad hoc network assuming alternative routes which keep the two nodes connected are node-disjoint. We provided a model for energy consumption of nodes during end-to-end packet transfer in two types of ad hoc networks: networks which support automatic repeat request (ARQ) to recover lost packets, and networks which do not support ARQ. On the basis of the energy consumption model, we analyzed the maximum duration that two nodes can keep communicating with each other in a static ad hoc network. We presented numerical algorithms which can predict at any moment the maximum duration that two nodes can still communicate with each other. Then, we derived a closed-form expression for the expected value of maximum node-to-node communication lifetime in static networks. We also derived upper and lower bounds on the lifetime of node-disjoint routes between two arbitrary nodes. Using extensive simulation studies, we verified the accuracy of our analysis. We also investigated node-to-node communication lifetime when there are concurrent communications between different pairs of source–destination nodes. We interestingly observed that if there are a few concurrent communications in the network, the maximum lifetime of node-to-node communication decreases linearly with the number of concurrent communications. Next, we plan to study node-to-node communication lifetime in mobile ad hoc networks.

Acknowledgment

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Appendix A

A.1. Proof of (10)

If a data packet is lost after M tries, no ACK will be transmitted for it. Hence, $\Pr\{\mathcal{Y}=0\}=(1-p)^M$. On the other hand, an ACK will be transmitted M times for a data packet, if the data packet is received correctly in each transmission try, but the transmitted ACK is lost in all $M-1$ previous tries. Hence, $\Pr\{\mathcal{Y}=M\}=p^M(1-q)^{M-1}$.

[3] Note that the simulation results show the average number of packets exchanged between a source and a destination.

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There could be two cases resulting in transmission of $1 \leqslant x \leqslant M-1$ ACKs for a single data packet. In the first case, the Mth transmission try of the data packet never happens, because the transmitter receives an ACK before reaching its maximum transmission tries. If this case, $x-1$ out of the first $m-1$, $x-1 \leqslant m \leqslant M-1$ transmission tries of a data packet could be successful, but all $x-1$ generated ACKs for them are failed. The mth transmission try of the data packet is successful, and the generated ACK for it (i.e., the xth transmitted ACK) is received successfully. The probability of this event is

$$E_1 = \sum_{m=x}^{M-1} \binom{m-1}{x-1} p^{x-1}(1-q)^{x-1}(1-p)^{m-1-(x-1)} pq = \sum_{m=x}^{M-1} \binom{m-1}{x-1} p^{x}(1-q)^{x-1}(1-p)^{m-x} q.$$

In the second case, the Mth transmission try of the data packet happens, because no ACK has been received during the last $M-1$ tries. Here, there are two subcases. In the first subcase, only $x-1$ out of $M-1$ transmission tries of the data packet are successful, but all $x-1$ transmitted ACKs are lost. The Mth transmission try of the data packet is also successful, which triggers transmission of the xth ACK. The probability of this event is

$$E_2 = \binom{M-1}{x-1} p^{x-1}(1-p)^{M-1-(x-1)}(1-q)^{x-1} p = \binom{M-1}{x-1} p^{x}(1-p)^{M-x}(1-q)^{x-1}.$$

In the second subcase, only x out of $M-1$ transmission tries of the data packet are successful, but all x generated ACKs for them are failed. However, the Mth transmission try fails, which prevents transmission of another ACK. The probability of this event is

$$E_3 = \binom{M-1}{x} p^{x}(1-p)^{M-1-x}(1-q)^{x}(1-p) = \binom{M-1}{x} p^{x}(1-p)^{M-x}(1-q)^{x}.$$

The probability of transmission $1 \leqslant x \leqslant M-1$ ACKs for a data packet is then $E_1 + E_2 + E_3$.

A.2. Proof of (44)

To prove (44), we need to show that

$$T_{s_m}\sigma(K^*) + \sum_{k=0}^{K^*-1} T_k[\sigma(k) - \sigma(k+1)] \geqslant T_{s_m}\theta(K^*) + \sum_{k=0}^{K^*-1} T_k[\theta(k) - \theta(k+1)]. \quad (47)$$

We can simplify (47) as

$$T_{s_m}[\sigma(K^*) - \theta(K^*)] + \sum_{k=0}^{K^*-1} T_k[\sigma(k) - \theta(k)] - \sum_{k=0}^{K^*-2} T_k[\sigma(k+1) - \theta(k+1)] \geqslant T_{K^*-1}[\sigma(K^*) - \theta(K^*)],$$

which can also be expressed as

$$\sum_{k=0}^{K^*-1} T_k \frac{\sigma(k) - \theta(k)}{\sigma(K^*) - \theta(K^*)} - \sum_{k=0}^{K^*-2} T_k \frac{\sigma(k+1) - \theta(k+1)}{\sigma(K^*) - \theta(K^*)} \geqslant T_{K^*-1} - T_{s_m}. \quad (48)$$

We replace k by $k+1$ in the second summation of (48), and merge the resulting summation with the first summation. Since $T_0 = 0$, (48) can equivalently be expressed as

$$\sum_{k=1}^{K^*-1} (T_k - T_{k-1}) \frac{\sigma(k) - \theta(k)}{\sigma(K^*) - \theta(K^*)} \geqslant T_{K^*-1} - T_{s_m}. \quad (49)$$

Therefore, if we show that (49) is true, recursively we can show that (47) is true as well. We notice that T_k is always greater than T_{k-1}. We also know that $\sigma(k) \geqslant \theta(k)$. Therefore, the right side of (49) is a positive value. Notice that K^* is the minimum number of routes required to prevent communication failure due to lack of routes. Hence, the source node dies after the nodes in the K^*th route. As a result, $T_{K^*-1} < T_{s_m}$. This means $T_{K^*-1} - T_{s_m} \leqslant 0$, which implies that (49) is true.

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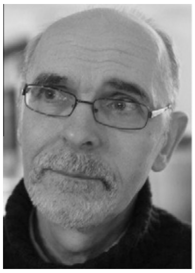
Javad Vazifehdan received his B.Sc. degree from Iran University of Science and Technology, Tehran, Iran in Electrical Engineering in 2002, and his M.Sc. degree from University of Tehran in Telecommunications in 2005. After 3 years successful work experience in development of real-time communication protocols for industrial automation, he jointed in 2007 the department of Telecommunications of Delft University of Technology, The Netherlands, to pursue his Ph.D. degree. He received his Ph.D. degree with honor (Cum Laude) in 2011. He is currently a post-doctoral researcher in Delft University of Technology. His main research area is energy-efficiency in wireless and mobile networks.

R Venkatesha Prasad received his bachelors degree in Electronics and Communication Engineering and M.Tech degree in Industrial Electronics from University of Mysore, India in 1991 and 1994. He received a Ph.D. degree in 2003 from Indian Institute of Science, Bangalore India. During 1996 he was working as a consultant and project associate for ERNET Lab of ECE at Indian Institute of Science. While pursuing the Ph.D. degree, from 1999 to 2003 he was also working as a consultant for CEDT, IISc, Bangalore for VoIP application developments as part of Nortel Networks sponsored project. In 2003 he was heading a team of engineers at the Esqube Communication Solutions Pvt. Ltd. Bangalore for the development of various real-time networking applications. Currently, he is a part time consultant to Esqube. From 2005 till date he is a senior researcher at Wireless and Mobile Communications group, Delft University of Technology working on the EU funded projects MAGNET/MAGNET Beyond and PNP-2008 and guiding graduate students. He is an active member of TCCN, IEEE SCC41, and reviewer of many IEEE Transactions and Elsevier Journals. He is on the TPC of many conferences including ICC, GlobeCom, ACM MM, ACM SIGCHI, etc. He is the TPC co-chair of CogNet workshop in 2007, 2008 and 2009 and TPC chair for E2Nets at IEEE ICC-2010. He is also running PerNets workshop from 2006 with IEEE CCNC. He is the Tutorial Co-Chair of CCNC 2009 and 2011 and Demo Chair of IEEE CCNC 2010.



Ignas Niemegeers got a degree in Electrical Engineering from the University of Ghent, Belgium, in 1970. In 1972 he received a M.Sc.E. degree in Computer Engineering and in 1978 a Ph.D. degree from Purdue University in West Lafayette, Indiana, USA. From 1978 to 1981 he was a designer of packet switching networks at Bell Telephone Mfg. Cy, Antwerp, Belgium. From 1981 to 2002 he was a professor at the Computer Science and the Electrical Engineering Faculties of the University of Twente, Enschede, The Netherlands. From 1995 to 2001 he was Scientific Director of the Centre for Telematics and Information Technology (CTIT) of the University of Twente, a multi-disciplinary research institute on ICT and applications. Since May 2002 he holds the chair Wireless and Mobile Communications at Delft University of Technology, where he is heading the Telecommunications Department. He was involved in many European research projects, e.g., the EU projects MAGNET and MAGNET Beyond on personal networks, EUROPCOM on UWB emergency networks and, eSENSE and CRUISE on sensor networks. He is a member of the Expert group of the European technology platform eMobility and IFIP TC-6 on Networking. His present research interests are 4G wireless infrastructures, future home networks, ad hoc networks, personal networks and cognitive networks. He has (co)authored close to 300 scientific publications and has coauthored a book on Personal Networks.

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