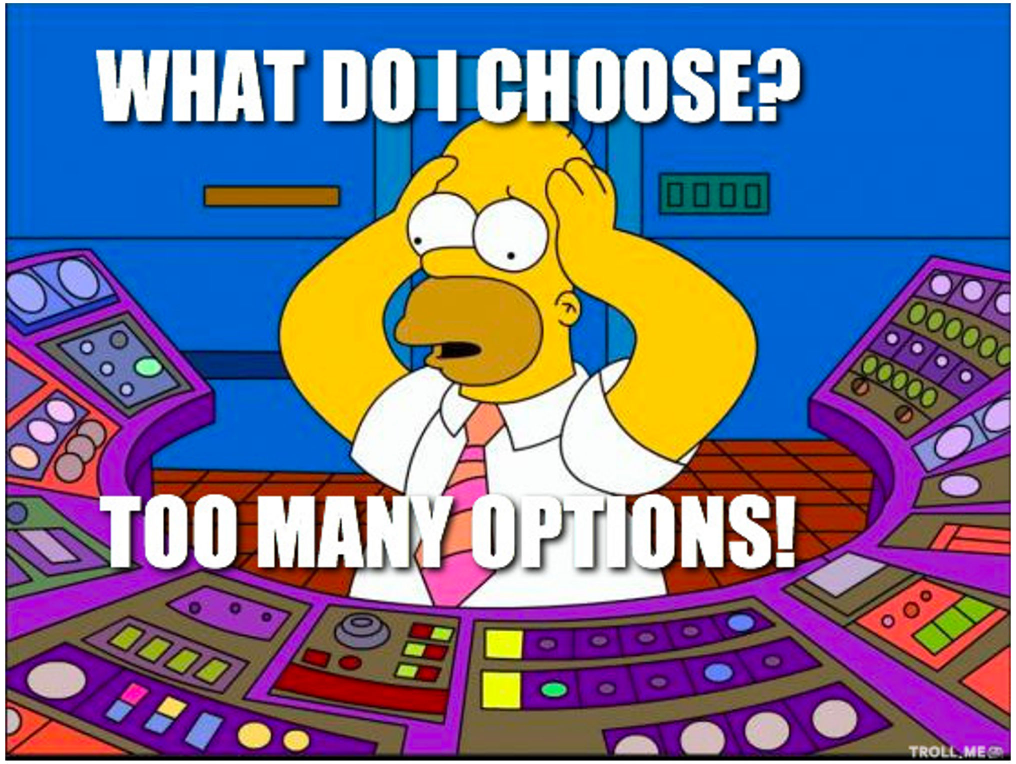
IN4390:Quantitative Performance Evaluation for Embedded Systems

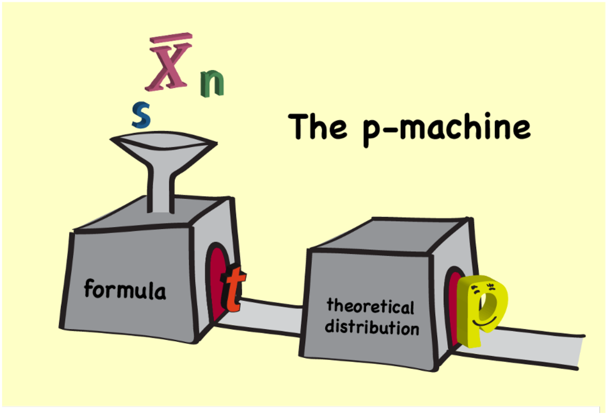
Lydia Y. Chen
y.chen-10@tudelft.nl

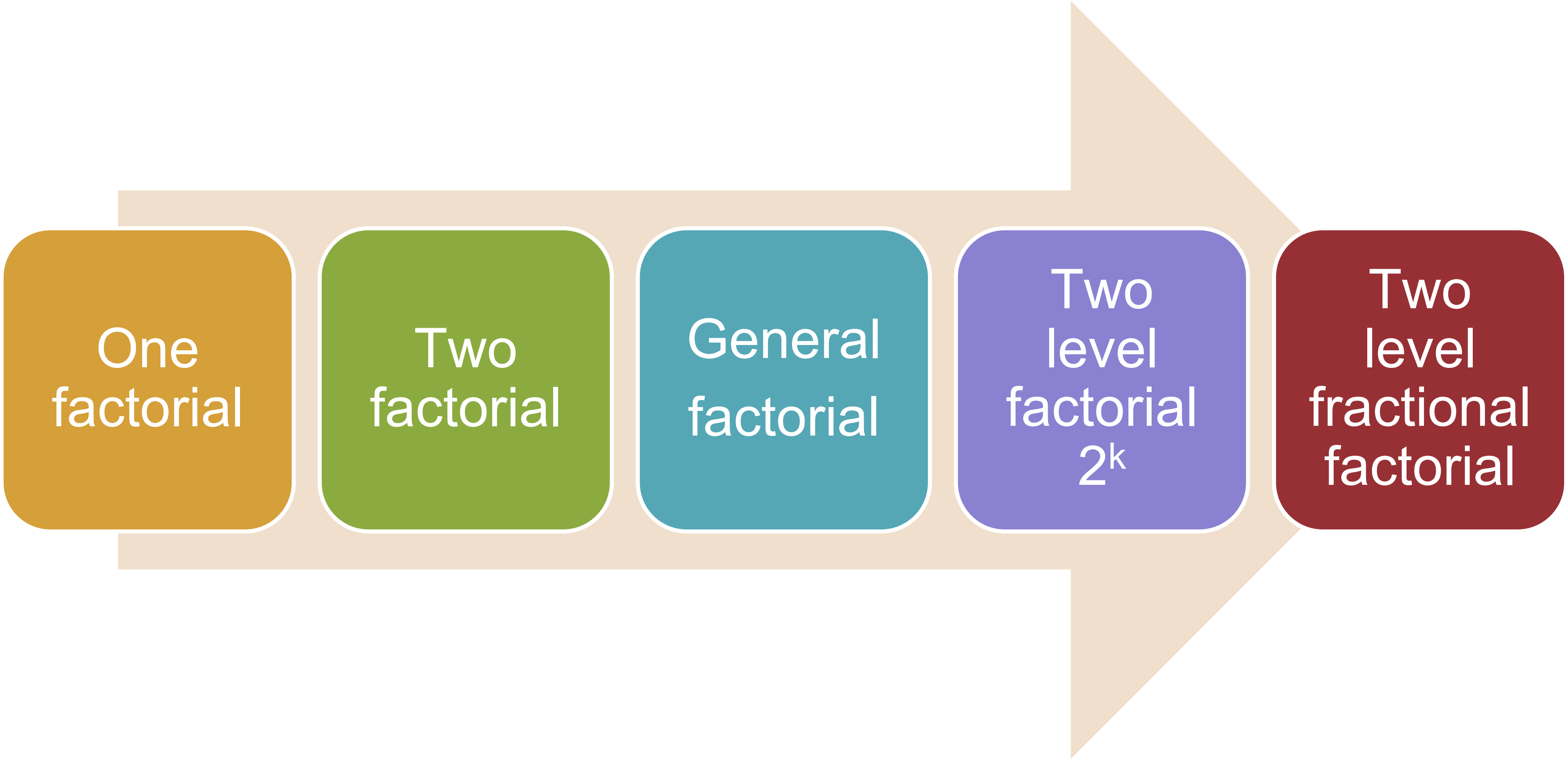
DoE is used for …..

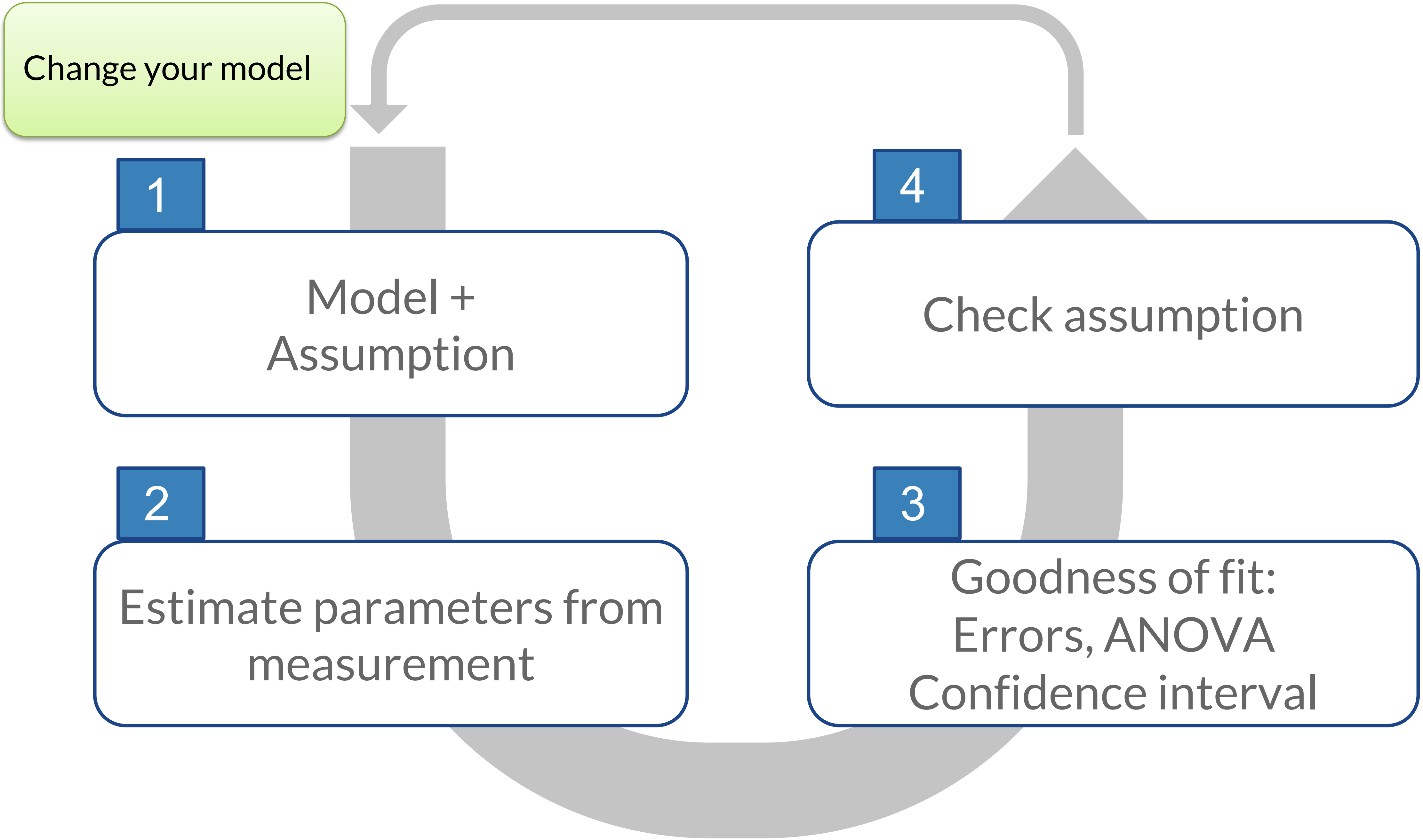


Design of Experiments II

- Two-factorial design
- General- factorial design




One factorial
Two factorial
General factorial
Two level factorial 2^k
Two level fractional factorial


Change your model
1
Model +
Assumption
2
Estimate parameters from
measurement
3
Goodness of fit:
Errors, ANOVA
Confidence interval
4
Check assumption

Effect Models

- One factor

$$y_{ij} = \mu + \alpha_j + e_{ij}$$

- Two factor

$$y_{ijk} = \mu + \alpha_j + \beta_i + \boxed{\gamma_{ij}} + e_{ijk}$$

Q: meaning index of i,j,k?

$$r_{ij} = \bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...}$$

- Suitable for categorical favors: e.g., high/low values, and different versions of software and hardware
- ANOVA implicitly assumes categorical variables

Regression Model

- When more than one factors are quantitative variables, regression models are handy, e.g.,

$$\hat{y} = b_0 + b_1 x_1$$

Q: What is the difference with effect models?

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + b_{12} x_1 x_2$$

Easier to see the impact of x.

- Estimating parameters by minimizing the sum of square errors $\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)^2$ with a constraint of $\sum_{i=1}^{n} e_i = \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i) = 0$

Q: What about distribution of e

Normal Distribution

Estimating Regression Parameters

- Taking derivative

$$\frac{d(SSE)}{d(b_0)} \quad \frac{d(SSE)}{d(b_1)} \quad \frac{d(SSE)}{d(b_2)} \ldots$$

- Computer software does all the fitting for you

2^k Factorial Designs

2^k Factorial Designs

- K factors, each at two levels which can be quantitative or qualitative
- Features: easy to analyze. It helps in sorting out impact of factors, and good if done at the beginning of a study
- Sign Table: specific way of coding levels of factors, a powerful tool to construct small number of experiments for models
- Regression model: response(y) is a function of factors (x)

Coding Scheme and Sign table

- We use (+1,-1) as the coded variables to denote high and low values of factor

Q: Size of sign table of 2^k

- For a 2^k design
  - The sign table has 2^k columns corresponding to parameters including intercept, main effects, two factor interactions…up to k-factor interactions.
  - It has 2^k rows, corresponding to experiment types
  - It can use to estimate 2^k parameters
  - For example k=2, say factor A and B
    - Main effect: A, B
    - Two factor interactions: AB
    - Model (regression): $y_{i,j} = q_0 + q_A x_{A_i} + q_B x_{B_i} + q_{AB} x_{A_i} x_{B_i} + e_{ij}$

$$\begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Q: what is k here?

K=2

Q: Write the model

Examples of 2^k factor experiment

What is the MIPS(millions instructions per second)given memory size and cache size? Two levels of memory sizes (A) and two level of cache sizes (B)

Performance in MIPS

Cache Size	Memory Size	
	4M Bytes	16M Bytes
1K		
2K		

Q: What are -1, and +1 here?

Memory Size-: +1 and -1 correspond to 4M and 16M bytes.
Cache Size-: +1 and -1 correspond to 1K and 2K respectively.

Sign Table of the Cache Example

$$x_A = \begin{cases} -1 & \text{if 4M bytes memory} \\ 1 & \text{if 16M bytes memory} \end{cases}$$

$$x_B = \begin{cases} -1 & \text{if 1K bytes cache} \\ 1 & \text{if 2K bytes cache} \end{cases}$$

Q: Why do you choose those values?

Rule of thumb: Choose values that are as far apart as possible.

Your First Sign Table: a matrix of size 2^k X 2^k

Step 1. Write the parameters on the top of the sign table

I	A	B	A B

Step 2. Fill out the signs of each column from main effects, then interaction effects

MATLAB

I	A	B	A B

Step 3. Run the experiments

Estimating Parameters for 2^2 design

- Case of no-replication
  - Model: $y = q_0 + q_A x_A + q_B x_B + q_{AB} x_A x_B$
  - Run exp according to the sign table.

Sign table encodes the parameter estimation

Experiment	q0	A (2^0)	B (2^1)	AB	Y
1	1	-1	-1	1	Y1=15
2	1	1	-1	-1	Y2=45
3	1	-1	1	01	Y3=25
4	1	1	1	1	Y4=75

Row view

Column view

A

Performance in MIPS

Cache Size	Memory Size	
	4M Bytes	16M Bytes
1K	15	45
2K	25	75

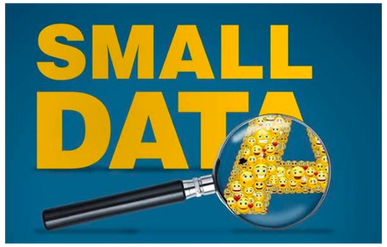
B

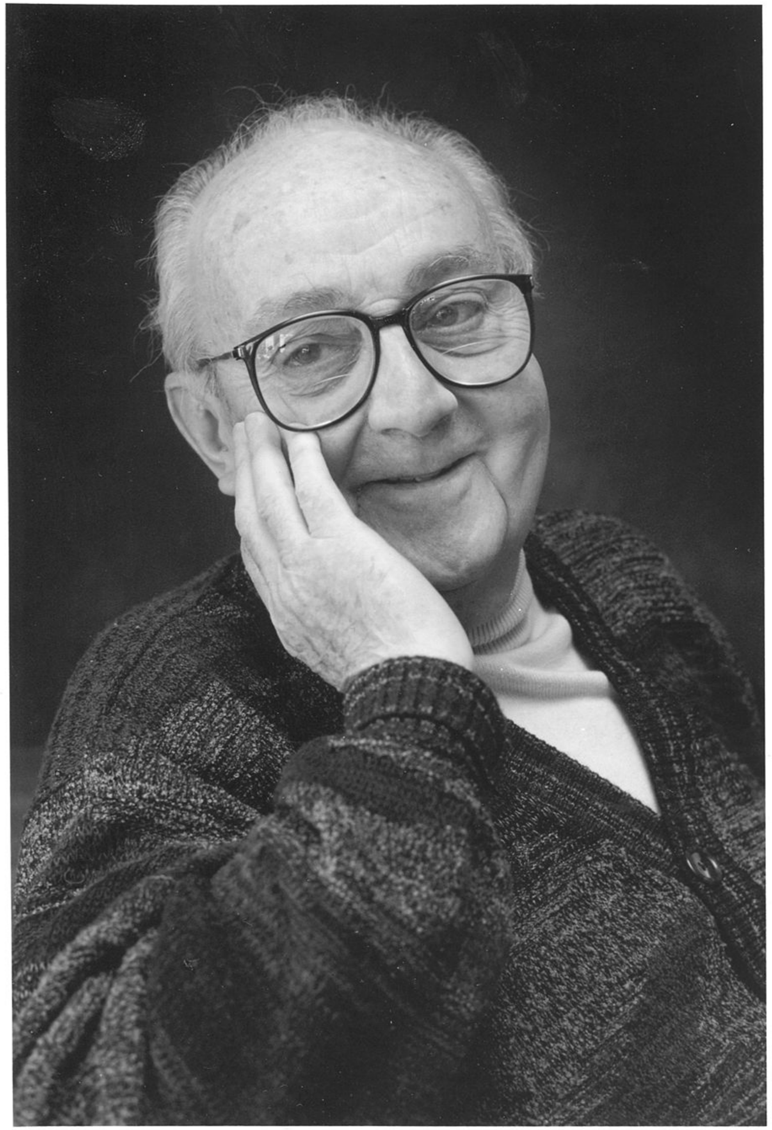
$$15 = q_0 - q_A - q_B + q_{AB}$$
$$45 = q_0 + q_A - q_B - q_{AB}$$
$$25 = q_0 - q_A + q_B + q_{AB}$$
$$75 = q_0 + q_A + q_B - q_{AB}$$

$$q_0 = \frac{1}{4}(y_1 + y_2 + y_3 + y_4)$$
$$q_A = \frac{1}{4}(-y_1 + y_2 - y_3 + y_4)$$
$$q_B = \frac{1}{4}(-y_1 - y_2 + y_3 + y_4)$$
$$q_{AB} = \frac{1}{4}(y_1 - y_2 - y_3 + y_4)$$

- Case of replication: replace y by average of y to estimate model parameters.

GEORGE BOX





Property of sign table

Exp	I	A	B	AB	Y
1	1	-1	-1	1	Y_1 (0)
2	1	1	-1	-1	Y_2 (a)
3	1	-1	1	-1	$Y_{3.}$ (b)
4	1	1	1	1	$Y_{4.}$ (ab)

- Observation
  - To estimate effect of Y_o, q_A $q_{B,}$ q_{AB} : product of corresponding column 0/A/B/AB and column Y_response, which is the average of all replications
  - Y_response : Product of row and vector [Y_0 , q_A, q_B, q_C]

Q: What are other properties?

- Theorems
  - Fractional factorial designs also use orthogonal vectors. $\sum_i x_{ij}x_{il} = 0. \forall j \neq l$
  - The sum of the products of any two columns is zero. $\sum_i x_{ij}^2 = 2^2, \forall j$
  - The sum of the squares of each column is 2^2.

Example of Memory-Cache Study

- What are the effects of A,B, AB?

Performance in MIPS

Cache Size	Memory Size	
	4M Bytes	16M Bytes
1K	$(15, 18, 12)$	$(25, 28, 19)$
2K	$(45, 48, 51)$	$(75, 78, 81)$

$$y_{i,j} = q_0 + q_{A_i} x_{A_i} + q_{B_i} x_{B_i} + q_{AB} x_{A_i} x_{B_i} + e_{ij}$$

$$q_0 = 41, q_A = 21.5, q_B = 9.5, q_{AB} = 5$$

$$y_{i,j} = 41 + 21.5 x_{A_i} + 9.5 x_{B_i} + 5 x_{A_i} x_{B_i} + e_{ij}$$

Q: Does that mean memory 1MB will lead to 21.5 MIPS?

N O! NO! Go back to natural variables

N O! NO! And, look at interaction term as well

$$X_{Ai} = \frac{X_{natural} - (low + high)/2}{(high - low)/2}$$

ANOVA for 2^2 design

Allocating variation to errors

- Error = Measured value- estimated values

$$\begin{aligned} e_{ij} &= y_{ij} - \hat{y}_i \\ &= y_{ij} - q_0 - q_A x_{Ai} - q_B x_{Bi} - q_{AB} x_{Ai} x_{Bi} \end{aligned}$$

- Total Variation

$$SST = \sum_{ij} (y_{ij} - \bar{y}_{..}) = SSA + SSB + SSAB + SSE$$

- F-Test
  - Hypothesis: Tested factors have no effect on the response y
  - Compute F0, and compare it with F tables with significance α

$$\frac{SSA/v_A}{SSE/v_e}, \frac{SSB/v_B}{SSE/v_e}, \frac{SSAB/v_{AB}}{SSE/v_e}$$

Q: Which factors could you test and how?

Allocation of Variation (ANOVA)

$$y_{i,j} = q_0 + q_A x_{A_i} + q_B x_{B_i} + q_{AB} x_{A_i} x_{B_i} + e_{ij}$$

- Total variation or total sum of squares:

$$SST = \sum_{ij} (y_{ij} - \bar{y}_{..})^2 = \sum_{ij} y_{ij}^2 - \sum_{ij} \bar{y}_{..}^2$$

➔SST=SSY-SS0

$$SST = \sum_{ij} (y_{ij} - \bar{y}_{..})^2 = 2^2 r q_A^2 + 2^2 r q_B^2 + 2^2 r q_{AB}^2 + \sum_{ij} e_{ij}^2$$

Derivation (I)

$$\sum_{ij} y_{ij} = \sum_{ij} q_0 + \sum_{i,j} q_A x_{A_i} + \sum_{i,j} q_B x_{B_i} + \sum_{i,j} q_{AB} x_{A_i} x_{B_i} + \sum_{ij} e_{ij}$$

$$\sum_{ij} y_{ij} = \sum_{ij} q_0 = 2^2 r q_0$$

$$\bar{y}_{..} = \frac{1}{2^2 r} \sum_{i,j} y_{i,j} = q_0$$

Allocation of Variation (ANOVA)

$$y_{i,j} = q_0 + q_A x_{A_i} + q_B x_{B_i} + q_{AB} x_{A_i} x_{B_i} + e_{ij}$$

- Total variation or total sum of squares:

$$SST = \sum_{ij} (y_{ij} - \bar{y}_{..})^2 = \sum_{ij} y_{ij}^2 - \sum_{ij} \bar{y}_{..}^2$$

$$SST = \sum_{ij} (y_{ij} - \bar{y}_{..})^2 = 2^2 r q_A^2 + 2^2 r q_B^2 + 2^2 r q_{AB}^2 + \sum_{ij} e_{ij}^2$$

➔SST=SSA+SSB+
SSAB+SSE

Derivation (II)

$$\sum_{ij} y_{ij}^2 = \sum_{ij} q_0^2 + \sum_{i,j} q_A^2 x_{A_i}^2 + \sum_{i,j} q_B^2 x_{B_i}^2 + \sum_{i,j} q_{AB}^2 x_{A_i}^2 x_{B_i}^2 + \sum_{ij} e_{ij}^2$$

$$\boxed{\begin{aligned} \mathrm{SSY} &= \mathrm{SS0} + \mathrm{SSA} + \mathrm{SSB} \\ &\quad + \mathrm{SSAB} + \mathrm{SSE} \end{aligned}}$$

Back to Example: Memory-Cache Study

To construct F-test of factors, we need to

- Degrees of freedom of SSA, SSB, SSAB, SSE, and SST

? 1 , 1, 1, 8, 11

- Compute F0 for all factors

$$\frac{SSA/1}{SSE/8}, \frac{SSB/1}{SSE/8}, \frac{SSAB/1}{SSE/8}$$

- Compare them F-statistics

Formula is so simple! You can do it by hand

$$\begin{aligned}
\mathrm{SSY} &= 15^2+18^2+12^2+45^2+\cdots+75^2+75^2+81^2 \\
&= 27204 \\
\mathrm{SS0} &= 2^2rq_0^2 = 12\times 41^2 = 20172 \\
\mathrm{SSA} &= 2^2rq_A^2 = 12\times(21.5)^2 = 5547 \\
\mathrm{SSB} &= 2^2rq_B^2 = 12\times(9.5)^2 = 1083 \\
\mathrm{SSAB} &= 2^2rq_{AB}^2 = 12\times 5^2 = 300 \\
\mathrm{SSE} &= 27204 - 2^2\times 3(41^2+21.5^2+9.5^2+5^2) \\
&= 102 \\
\mathrm{SST} &= \mathrm{SSY}-\mathrm{SS0} \\
&= 27204-20172 = 7032
\end{aligned}$$

Example of computer output

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_q	*P*-Value
A	208.33	1	208.33	53.15	0.0001
B	75.00	1	75.00	19.13	0.0024
AB	8.33	1	8.33	2.13	0.1826
Error	31.34	8	3.92		
Total	323.00	11			

Q: How to interpret ?

Q: Graph interpretation ?

Given significance=0.05, factor A. B, are significant, and factor AB is not significant

Confidence Intervals for 2² Experiment

$$q_i \mp t_{[1-\alpha/2;2^2(r-1)]} s_{q_i}$$

Q: The difference between F-test?

It's for each parameter, whereas F is for the significance of overall factors

Derivation

- Assumption

$$\text{error} \sim \mathcal{N}(0, \sigma_e) \qquad \text{y} \sim \mathcal{N}(\bar{y}, \sigma_e)$$

- Estimate of variance

$$\hat{\sigma_e}^2 = s_e^2 = \frac{1}{2^2(r-1)} \sum_{ij} e_{ij}^2 = \frac{SSE}{2^2(r-1)}$$

  - Denominator = $2^2(r-1)$ = # of independent terms in SSE,SSE has $2^2(r-1)$ degrees of freedom.
  - Similarly, $s_{q_A} = s_{q_B} = s_{q_{AB}} = \frac{s_e}{\sqrt{2^2 r}}$

Back to Memory-Cache Example

- For Memory-cache study: Standard deviation of errors and factors

$$s_e = \sqrt{\frac{\text{SSE}}{2^2(r-1)}} = \sqrt{\frac{102}{8}} = \sqrt{12.75} = 3.57 \qquad s_{q_i} = s_e/\sqrt{(2^2 r)} = 3.57/\sqrt{12} = 1.03$$

- For 90% Confidence: check with $t_{[0.95,8]} = 1.86$

Q: Why 0.95?

Performing two tailed T test.

- Confidence intervals:
  - $q_0 = (39.08, 42.91)$
  - $q_A = (19.58, 23.41)$
  - $q_B = (7.58, 11.41)$
  - $q_{AB} = (3.08, 6.91)$

Q: Are they relevant?

Yes all relevant i.e excluding 0 in their intervals.

General $2^k r$ Factorial Design

- K factors at two levels each.
- 2^k experiments by a sign table
- 2^k effects:

$$\text{k main effects}$$
$$\binom{k}{2}\text{two factor interactions}$$
$$\binom{k}{3}\text{three factor interactions...}$$

Q: Example of 3 factors, A, B, C and sign table

3 factors, A.B, C
Experiments: 2^3
Effects: A, B, C, AB, AC, BC, ABC

	I	A	B	C	AB	AC	BC	ABC
Exp 1	1	-1	-1	-1	1	1	1	-1
Exp 2	1	1	-1	-1	-1	-1	1	1
Exp 3	1	-1	1	-1	-1	1	-1	1
Exp 4	1	1	1	-1	1	-1	-1	-1
Exp 5	1	-1	-1	1	1	-1	-1	1
Exp 6	1	1	-1	1	-1	1	-1	-1
Exp 7	1	-1	1	1	-1	-1	1	-1
Exp 8	1	1	1	1	1	1	1	1

General $2^k r$ Factorial Design

- Model:

Q: What are index i and j?

$$y_{ij} = q_0 + q_A x_{Ai} + q_B x_{Bi} + q_{AB} x_{Ai} x_{Bi} + \cdots + e_{ij}$$

- Parameter estimation:

$$q_j = \frac{1}{2^k} \sum_{i=1}^{2^k} S_{ij} \bar{y}_i$$

MATLAB

Q: Example of 3 factors model and SST decomposition ?

S_{ij} = (i,j)th entry in the sign table

- ANOVA

$$\mathrm{SSY} = \sum_{i=1}^{2^k} \sum_{j=1}^{r} y_{ij}^2$$
$$\mathrm{SS0} = 2^k r q_0^2$$
$$\mathrm{SST} = \mathrm{SSY} - \mathrm{SS0}$$
$$\mathrm{SS}j = 2^k r q_j^2 \mathrm{j} = 1, 2, \ldots, 2^k - 1$$

$$SST = SSY - SSO = \sum_j SSj + SSE$$

$$y_{i,j} = q_0 + q_A x_{A_i} + q_B x_{B_i} + q_B x_{C_i} + q_{AB} x_{A_i} x_{B_i} + q_{AC} x_{A_i} x_{C_i} + q_{BC} x_{B_i} x_{C_i} q_{ABC} x_{A_i} x_{B_i} + x_{A_i} + e_{ij}$$

$$SST = SSA + SSB + SSC + SSAB + SSAC + SSBC + SSABC$$

General $2^k r$ Factorial Design

- Percentage of y's variation explained by ***j*** th effect =

$$(\mathrm{SS}j/\mathrm{SST}) \times 100\%$$

- Standard deviation of errors:

$$s_e = \sqrt{\frac{\mathrm{SSE}}{2^k(r-1)}}$$

- Standard deviation of effects:

$$s_{q_0} = s_{q_A} = s_{q_B} = s_{q_{AB}} = s_e/\sqrt{2^k r}$$

2^3 Design Example

Three factors in designing a machine:

- Cache size
- Memory size
- Number of processors

	Factor	Level -1	Level 1
A	Memory Size	4MB	16MB
B	Cache Size	1kB	2kB
C	Number of Processors	1	2

1. Generate the design and run experiments.

I	A	B	C	AB	AC	BC	ABC	
1	-1	-1	-1	1	1	1	-1	(
1	1	-1	-1	-1	-1	1	1	(
1	-1	1	-1	-1	1	-1	1	(
1	1	1	-1	1	-1	-1	-1	(
1	-1	-1	1	1	-1	-1	1	(
1	1	-1	1	-1	1	-1	-1	(
1	-1	1	1	-1	-1	1	-1	(
1	1	1	1	1	1	1	1	(

2. Analyze the model fitting: ANOVA and CIs.

2^3 Design Example

Component	Sum of Squares	Percent Variation
y	4.9E4	
$\bar{y}$	3.8E4	
y-$\bar{y}$	1.1E4	100.00%
A	1683.0	14.06%
B	693.3	5.79%
C	9009.0	75.27%
AB	198.3	1.66%
AC	135.4	1.13%
BC	84.4	0.70%
ABC	0.4	0.00%
Errors	164.0	1.37%

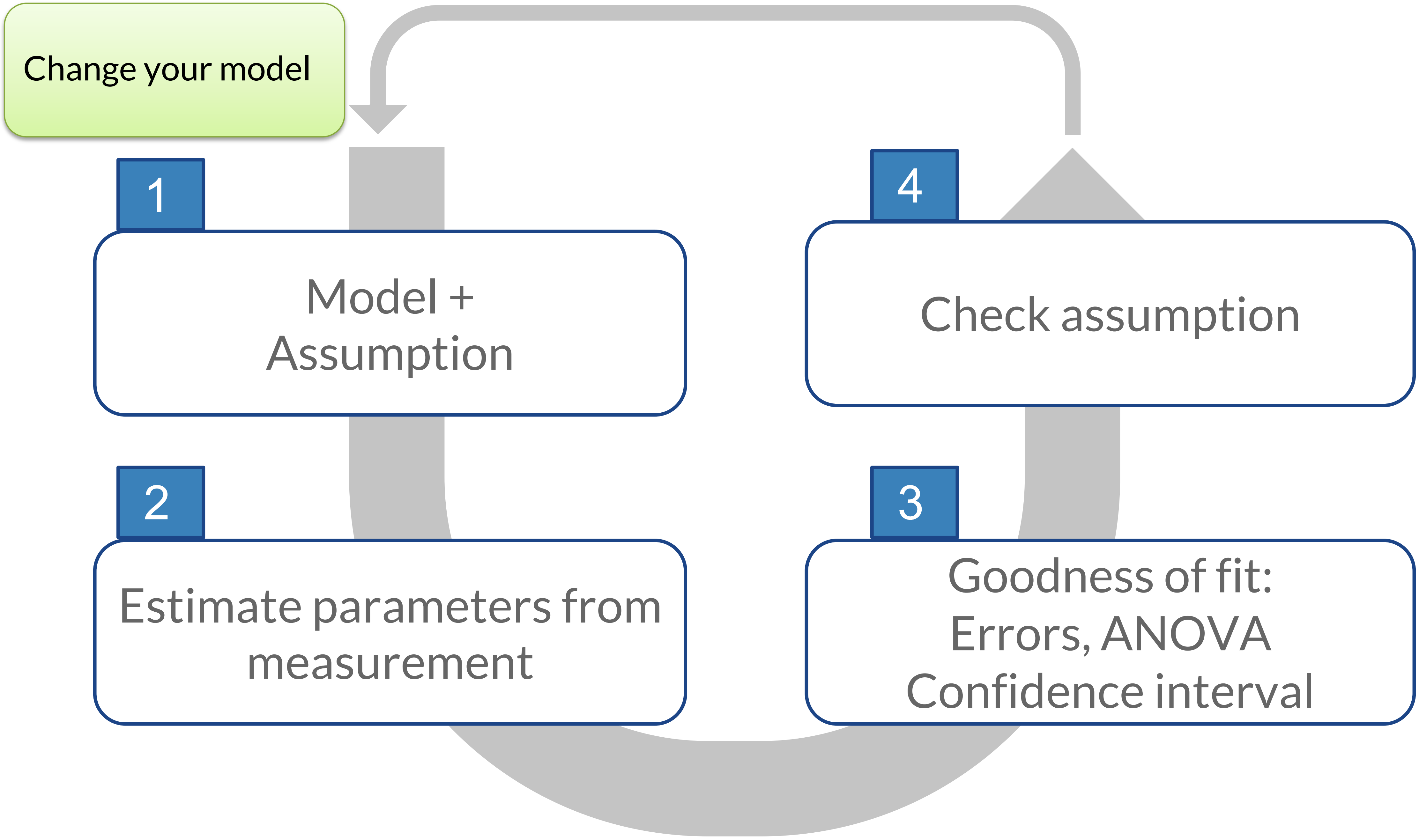
$$
\begin{aligned}
q_0 &= (39.00, 40.74) \\
q_A &= (7.50, 9.25) \\
q_B &= (4.50, 6.25) \\
q_C &= (18.50, 20.24) \\
q_{AB} &= (2.00, 3.75) \\
q_{AC} &= (1.50, 3.25) \\
q_{BC} &= (1.00, 2.75) \\
q_{ABC} &= (-1.00, 0.75)
\end{aligned}
$$

Q: Effects important ?

Q: Parameter significant?

Q: Is this a good model? Any suggestion?

New model: contain only the main effects and second order interactions


Change your model
1
Model +
Assumption
2
Estimate parameters from
measurement
3
Goodness of fit:
Errors, ANOVA
Confidence interval
4
Check assumption

2^{k-p} Fractional Factorial Designs

Supplemental

2^{k-p} Fractional Factorial Designs

- Large number of factors
  - large number of experiments
  - full factorial design too expensive
  - Use a fractional factorial design
- 2^{k-p} design allows analyzing k factors with only 2^{k-p} experiments.
  - 2^{k-1} design requires only ???? as many experiments 1/2
  - 2^{k-2} design requires only ???? of the experiments 1/4

Q: How to generate those experiments?

MATLAB

Understanding the sign table

- Design Table of 2^{7-4} generated by by DoE
- Study 7 factors with only 8 experiments!

Expt No.	A	B	C	D	E	F	G
1	-1	-1	-1	1	1	1	-1
2	1	-1	-1	-1	-1	1	1
3	-1	1	-1	-1	1	-1	1
4	1	1	-1	1	-1	-1	-1
5	-1	-1	1	1	-1	-1	1
6	1	-1	1	-1	1	-1	-1
7	-1	1	1	-1	-1	1	-1
8	1	1	1	1	1	1	1

Q: What's the maximum number of parameter can we learn from these 8 experiments?

8

Q: Which parameters are included in the model

Q: How to decide?

Generator function

Fractional Design Features

- Full factorial design is easy to analyze due to orthogonality of sign vectors.
  - Fractional factorial designs also use orthogonal vectors. The sum of each column is zero.

$$\sum_i x_{ij} = 0; \forall j$$

  - The sum of the products of any two columns is zero.

$$\sum_i x_{ij} x_{il} = 0, \forall j \neq l$$

  - The sum of the squares of each column is 2^{7-4}.

$$\sum_i x_{ij}^2 = 2^{7-4}, \forall j$$

Expt No.	A	B	C	D	E	F	G
1	-1	-1	-1	1	1	1	-1
2	1	-1	-1	-1	-1	1	1
3	-1	1	-1	-1	1	-1	1
4	1	1	-1	1	-1	-1	-1
5	-1	-1	1	1	-1	-1	1
6	1	-1	1	-1	1	-1	-1
7	-1	1	1	-1	-1	1	-1
8	1	1	1	1	1	1	1

Confounding

Confounding: Only the combined influence of two or more effects can be computed. For example

Expt No.	A	B	C	AB	AC	BC	D	
1	-1	-1	-1	1	1	1	-1	y_1
2	1	-1	-1	-1	-1	1	1	y_2
3	-1	1	-1	-1	1	-1	1	y_3
4	1	1	-1	1	-1	-1	-1	y_4
5	-1	-1	1	1	-1	-1	1	y_5
6	1	-1	1	-1	1	-1	-1	y_6
7	-1	1	1	-1	-1	1	-1	y_7
8	1	1	1	1	1	1	1	y_8

$$\boxed{q_A = \sum_i y_i x_{Ai} = \frac{-y_1 + y_2 - y_3 + y_4 - y_5 + y_6 - y_7 + y_8}{8}}$$

$$q_D = \sum_i y_i x_{Di} = \frac{-y_1 + y_2 + y_3 - y_4 + y_5 - y_6 - y_7 + y_8}{8}$$

$$q_{ABC} = \sum_i y_i x_{Ai} x_{Bi} x_{Ci} = \frac{-y_1 + y_2 + y_3 - y_4 + y_5 - y_6 - y_7 + y_8}{8}$$

$$q_{ABC} = q_D$$

Q: ABC is confounded with D, Problem?

Q:More confounding? How to find?

Algebra of Confounding

- Given just one confounding, it is possible to list all other confoundings.
- Rules:
  - I is treated as unity
  - Any term with a power of 2 is erased,

- Example. I=ABCD, multiplying A on both sides

  $\rightarrow IA = A^2BCD$

  $\rightarrow A = BCD$

- Example : Multiplying both sides by B,C,D and AB

$$B = AB^2CD = ACD$$
$$C = ABC^2D = ABD$$
$$D = ABCD^2 = ABC$$
$$AB = A^2B^2CD = CD$$

Q: Exercise

In a 2^{k-p} design, 2^p effects are confounded together

A Fractional Factorial Design is Not Unique.

- It has 2^p different designs.
- For example, 2^{4-1} has following 2^1 designs

$I = ABCD$

Expt No.	A	B	C	AB	AC	BC	D
1	-1	-1	-1	1	1	1	-1
2	1	-1	-1	-1	-1	1	1
3	-1	1	-1	-1	1	-1	1
4	1	1	-1	1	-1	-1	-1
5	-1	-1	1	1	-1	-1	1
6	1	-1	1	-1	1	-1	-1
7	-1	1	1	-1	-1	1	-1
8	1	1	1	1	1	1	1

Another 2^{4-1} Experimental Design

Expt No.	A	B	C	D	AC	BC	ABC
1	-1	-1	-1	1	1	1	-1
2	1	-1	-1	-1	-1	1	1
3	-1	1	-1	-1	1	-1	1
4	1	1	-1	1	-1	-1	-1
5	-1	-1	1	1	-1	-1	1
6	1	-1	1	-1	1	-1	-1
7	-1	1	1	-1	-1	1	-1
8	1	1	1	1	1	1	1

$I = ABD$

Q: Which one is better?

Resolution will tell us.

Design Resolution

- Resolution: tells us the order of effects that are confounded
  - Def 1: A design of resolution R if no p-factor effect is aliased/confounded with another other effect containing less than R-p factor
  - Def 2: **Resolution of a Design = Minimum orders of confoundings**
    - Order of an effect = Number of terms. E.g., Order of I=***ABCD*** = 4 because order of I = 0
    - Order of a confounding = Sum of order of two terms. E.g., AB=CDE is of order ??? 5

Q: Example of 2^{3-1} with I=ABC, What resolution?

A=BC, B=CD, C=AB Resolution III

Q: A Better design: higher or lower resolution?

Higher the resolution is better!

Back to the Example of 2^{4-1}

Q: What are their resolutions?

- Compare these two designs of 2^{4-1}.

$I = ABCD$

A=BCD,B=ACD,
C=ABD,AB=CD,AC=BD,BC=AD,ABC=D, I=ABCD

Resolution IV

Q: Winner is?

$I = ABD$

A=BD,B=AD,C=ABCD
AB=D,AC=BCD,AD=AB,BC=ACD,ABC=CD, I=ABD

Resolution III

Success of fractional factorial design is based on

- The sparsity of effects principles
- The projection property
- Sequential experimentation

Check out “Design and Analysis of Experiments ” from Douglas Montgomery

Recommended Readings

- The Art of Computer Systems Performance Analysis: Chapter 18, and 19

- Design and Analysis of Experiments: Chapter 6 and 8

Thanks!

Any questions?

lydiaychen@ieee.org

Ychen-10@tudelft.nl