TECHNISCHE UNIVERSITEIT DELFT
Faculteit Elektrotechniek, Wiskunde en Informatica

IN4390 – Quantitative Evaluation of Embedded Systems

January 24th, 2022, from 09:00 to 12:00

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Question:	1	2	3	4	5	6	7	Total
Points:	13	6	6	20	15	15	15	90
Score:								

- This is a closed book exam
- You may use a **simple** calculator only (i.e. graphical calculators are not permitted)
- Write your answers with a black or blue pen, not with a pencil
- **Always justify your answers, unless stated otherwise**

- The exam covers the following material:
  (a) the paper "Basic Concepts and Taxonomy of Dependable and Secure Computing" by A. Avizienis ; J.-C. Laprie ; B. Randell ; C. Landwehr
  (b) chapters 18-20,22-23 (DoE), and 30-33 (Queueing Theory) of the book "The Art of Computer Systems Performance Analysis" by R. Jain
  (c) the paper "Petri nets: Properties, analysis and applications" by T. Murata
  (d) chapters 11.2 (DTMC), and 11.3 (CTMC) of the book "Introduction to probability, statistics, and random processes" by H. Pishro-Nik

IN4390 QEES *Cheat Sheet*

Operational Laws	
Utilization law	$U = XS$
Little's law	$N = XR$
Forced-flow law	$X_k = V_k X$
Bottleneck law	$U_k = D_k X$
Operational Bounds	
Througput	$X \leq \min\left(\frac{1}{D_{max}}, \frac{N}{D+Z}\right)$
Response time	$R \geq \max\left(D, N \times D_{max} - Z\right)$
Queueing Theory M/M/1	
Utilization	$U = XS = \lambda/\mu = \rho$
Probability of n clients in the system	$P_n = \rho^n(1-\rho)$
Mean #clients in the system	$N = \rho/(1-\rho) = \lambda/(\mu-\lambda)$
Mean #clients in the queue	$N_Q = N - \rho$
Mean response time	$R = N/\lambda = 1/(\mu-\lambda)$
Mean waiting time	$W = R - S = \rho/(\mu-\lambda)$
Basic Math	
Geometric series	$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$, for $\lvert r\rvert < 1$

ANOVA Table for One Factor Experiments

Component	Sum of Squares	%Variation	DF	Mean Square	F-Comp.	F-Table
y	SSY=$\sum y_{ij}^2$		ar			
$\bar{y}_{..}$	SS0=$ar\mu^2$		1			
y-$\bar{y}_{..}$	SST=SSY-SS0	100	ar-1			
A	SSA = $r\Sigma\ \alpha_i^2$	$100\left(\frac{\text{SSA}}{\text{SST}}\right)$	a-1	MSA = $\frac{\text{SSA}}{a-1}$	$\frac{\text{MSA}}{\text{MSE}}$	$F\ [1-\alpha; a-1, a(r-1)]$
e	SSE=SST- SSA	$100\left(\frac{\text{SSE}}{\text{SST}}\right)$	$a(r-1)$	MSE = $\frac{\text{SSE}}{a(r-1)}$		

Question 1 [13 points]

Measuring is at the basis of performance evaluation. Data obtained from experiments allows validating abstract models of the system under study. As not everything is modeled and/or under control, repeated measurements of a single experiment will result in (slightly) different numbers. Averaging these numbers is a crude – but often convenient – way to summarize them.

(a) [2 points] Explain when it is better to use the median than the average (mean).

(b) [2 points] Mention a downside of using the median.

(c) [2 points] Mention a data distribution where both the mean and the median do not provide meaningful information. How does the alternative "solution" of reporting the mean and variance do in this case?

When designing an Embedded System, often multiple performance metrics are to be considered. Having multiple objectives complicates the exploration of design alternatives as the different objectives may conflict, that is, objective 1 may be better in configuration A, while configuration B excels in objective 2.

(d) [2 points] Describe what a Pareto front denotes.

(e) [5 points] Give an multi-objective example – from the ES domain – that highlights the issue, and report the corresponding Pareto front.

Question 2 [6 points]

A two-factor ANOVA table is obtained through a factorial design.

v	DF	SS	MS	F
A	**	150	37.50	15
B	1	100	**	**
Interaction	**	**	**	**
Error	**	50	2.50	
Total	29	380		

(a) [1 point] What is the degree of freedom of the error term?

(b) [1 point] many replicates of experiments were performed?

(c) [2 points] What are the F statistics for factor B, and factor AB (interaction)?

(d) [2 points] Which factors are significant given the following Table of significance level of 5%.

	$F_{1,5}$	$F_{2,5}$	$F_{3,5}$	$F_{4,5}$	$F_{5,5}$	$F_{1,10}$	$F_{2,10}$	$F_{3,10}$
Values	6.61	5.78	5.41	5.19	5.05	4.96	4.10	3.71

	$F_{4,10}$	$F_{5,10}$	$F_{1,20}$	$F_{2,20}$	$F_{3,20}$	$F_{4,20}$	$F_{5,20}$	
Values	3.47	3.32	4.35	3.49	3.09	2.86	2.71	

Question 3 [6 points]

An 8-run experiment was performed to figure out the important system parameters to run an image classifier on mobile phones. Based on the following sign table, answer the questions below:

Run	CPU speed [GHZ] (A)	Memory size[GB] (B)	Classier sizes [MB] (C)	Memory size [KB](D)
1	2.1	1	30	10
2	4.2	1	30	50
3	2.1	2	30	50
4	4.2	2	30	10
5	2.1	1	60	50
6	4.2	1	60	10
7	2.1	2	60	10
8	4.2	2	60	50

(a) [1 point] What kind of design is this?

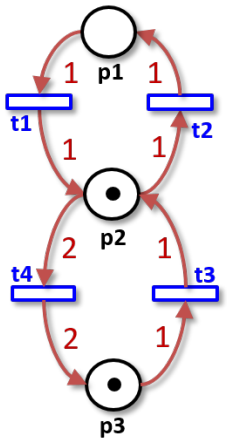
(b) [2 points] What is the generator function?

(c) [1 point] Which factor is confounded with factor A, B, C, D respectively?

(d) [2 points] If the generator function I=ABD is used, which other factor will be founded with factor A, B, C, and D, respectively? Is this a better solution?

Question 4 [20 points]

(a) Consider the following Petri Net:



i. [4 points] Draw the reachability or the coverability graph of this Petri Net.

ii. [2 points] Is this Petri Net k-bounded for some positive integer k? If yes, what is the minimum value of k? If no, explain your reasoning.

(b) i. [4 points] Draw the Petri Net that is associated with the following incidence matrix:

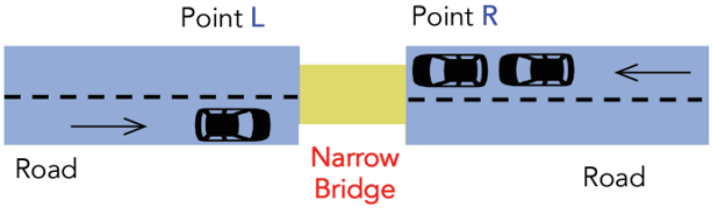
$$A = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

ii. [2 points] If the initial marking m_0 is as follows:

$$m_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

and **t1** (corresponding to first column of A) fires, calculate analytically, i.e., using algebraic equations, what will be the obtained marking m_1. Please include the calculations in your answer.

(c) [8 points] Consider the two-way road in the figure below, where the cars need to cross a narrow bridge that has only one lane. This means that, at each time, only one car can cross the bridge between points L and R, and the other cars have to wait at the respective sides (Point L and Point R). Please draw a Petri Net that models this road system and gives priority to the cars waiting at Point L, i.e., it prioritizes the crossing from L to R.



Question 5 [15 points]

A computer card game has been designed and you are asked to prove the correctness of the algorithm used to shuffle a deck of n cards, where $n = 52$. The algorithm works as follows. Starting with any initial ordering of the cards, one of the numbers $(1, 2, ..., 52)$ is chosen at random and with equal probability. If number i is chosen, we move the card from position i in the deck to the top, i.e. to position 1. We repeatedly perform the same operation.

In the sequel below you will show that, in the limit, the deck is perfectly shuffled in the sense that the resultant ordering is equally likely to be any of the $n!$ possible orderings.

(a) [1 point] To model this problem as a Discrete-Time Markov Chain, how would you define the random process X_n and the states that X_n can take? How many states do you have?

(b) [1 point] Do all states have the same number of incoming and outgoing links? Explain your reasoning.

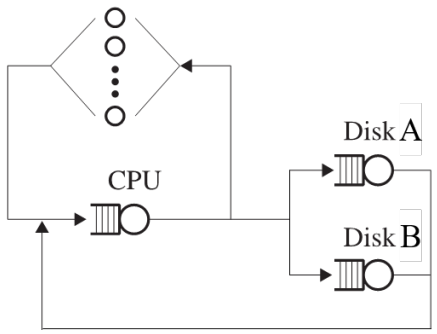
(c) [3 points] Is the resulting Markov Chain irreducible and aperiodic? Explain your reasoning.

(d) [7 points] Prove that the shuffling algorithm is correct by showing that the limiting distribution of the DTMC leads to the uniform distribution (i.e. $1/n!$).
Hint: start your proof using the equation: $\pi P = \pi$ and show that $\pi_i = 1/n!$ for all i.

(e) [3 points] Considering a deck of 3 cards, with cards A,B,C, and assuming that X_0 starts with the ordering (A,B,C), how many steps are required to have a non-zero probability of reaching all other possible orderings (i.e. states)? Explain your reasoning.

Question 6 [15 points]

Consider a timesharing system with two disks (A and B) configured as follows:



Through an hour-long measurement, the probabilities for jobs completing the service at the CPU were found to be 0.75 to disk A, 0.15 to disk B, and 0.1 to the terminals. The user think time was clocked to be 5 seconds, the disk service times are 30 milliseconds and 25 milliseconds, while the average service time per visit to the CPU was 40 milliseconds.

(a) [4 points] For each job, what are the visit ratios for the CPU, disk A, and disk B?
Hint: Argue that $V_{CPU} = 1 + V_A + V_B$.

(b) [3 points] For each device, what is the (total) service demand?

(c) [3 points] If disk A's utilization is 50%, what is the utilization of the CPU and disk B?

(d) [2 points] What is the average response time when there are 20 users on the system?

(e) [3 points] What is the best possible response time if one may upgrade one component (CPU or disk)?

Question 7 [15 points]

At a large hotel, taxi cabs arrive at a rate of 15 per hour, and passengers arrive at the rate of 12 per hour. Whenever taxi cabs are waiting, passengers are served immediately upon arrival (one per taxi). Whenever passengers are waiting, taxi cabs are loaded immediately upon arrival. A maximum of three cabs can wait at a time (other cabs must go elsewhere).

(a) [5 points] Find an appropriate way to model this queueing system as an M/M/1 queue; clearly state what information is encoded in a state, and draw the corresponding state transition diagram.

(b) [3 points] Calculate (i) the expected number of cabs waiting and (ii) the expected number of passengers waiting.

(c) [4 points] Calculate the average waiting times of (i) cabs and (ii) passengers.

(d) [3 points] What would be the impact of allowing four cabs to wait at a time? Compute the average number of taxis and passengers waiting in this case.