

**IN4390 – Quantitative Evaluation of Embedded Systems**

April 3rd, 2020, from 09:00 to 12:00

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Question:	1	2	3	4	5	6	Total
Points:	10	5	20	15	15	15	80
Score:							

- This is a closed book exam
- You may use a **simple** calculator only (i.e. graphical calculators are not permitted)
- Write your answers with a black or blue pen, not with a pencil
- **Always justify your answers, unless stated otherwise**
- The exam covers the following material:
  - (a) the paper “Basic Concepts and Taxonomy of Dependable and Secure Computing” by A. Avizienis ; J.-C. Laprie ; B. Randell ; C. Landwehr
  - (b) chapters 18-20,22-23 (DoE), and 30-33 (Queueing Theory) of the book “The Art of Computer Systems Performance Analysis” by R. Jain
  - (c) the paper “Petri nets: Properties, analysis and applications” by T. Murata
  - (d) chapters 11.2 (DTMC), and 11.3 (CTMC) of the book “Introduction to probability, statistics, and random processes” by H. Pishro-Nik
  - (e) the paper “Exploring Exploration: A Tutorial Introduction to Embedded Systems Design Space Exploration” by A.D. Pimentel

<b>Operational Laws</b>  Utilization law Little's law Forced-flow law Bottleneck law	$U = XS$ $N = XR$ $X_k = V_k X$ $U_k = D_k X$
<b>Operational Bounds</b>  Througput Response time	$X \leq \min \left( \frac{1}{D_{max}}, \frac{N}{D + Z} \right)$ $R \geq \max (D, N \times D_{max} - Z)$
<b>Queueing Theory M/M/1</b>  Utilization Probability of $n$ clients in the system Mean #clients in the system Mean #clients in the queue Mean response time Mean waiting time	$U = XS = \lambda/\mu = \rho$ $P_n = \rho^n (1 - \rho)$ $N = \rho/(1 - \rho) = \lambda/(\mu - \lambda)$ $N_Q = N - \rho$ $R = N/\lambda = 1/(\mu - \lambda)$ $W = R - S = \rho/(\mu - \lambda)$
<b>Basic Math</b>  Geometric series	$\sum_{k=0}^{\infty} r^k = \frac{1}{1 - r}, \text{ for }  r  < 1$

## ANOVA Table for One Factor Experiments

Compo- nent	Sum of Squares	%Variation	DF	Mean Square	F- Comp.	F- Table
y	$SSY = \sum y_{ij}^2$		ar			
$\bar{y}_{..}$	$SS0 = ar\mu^2$		1			
$y - \bar{y}_{..}$	$SST = SSY - SS0$	100	ar-1			
A	$SSA = r \sum \alpha_i^2$	$100 \left( \frac{SSA}{SST} \right)$	a-1	$MSA = \frac{SSA}{a-1}$	$\frac{MSA}{MSE}$	$F_{[1 - \alpha; a - 1, a(r - 1)]}$
e	$SSE = SST - SSA$	$100 \left( \frac{SSE}{SST} \right)$	$a(r - 1)$	$MSE = \frac{SSE}{a(r-1)}$		

## Question 1

[10 points]

A two-factor ANOVA table is obtained through a factorial design.

Two-way ANOVA: y versus A, B					
Source	DF	SS	MS	F	P
A	1		0.002		0.998
B		180.375			0.093
Interaction	3	8.479			0.932
Error	8				
Total	15	347.653			

- (a) 2 points How many levels were used for factor B?
- (b) 2 points How many replicates of the experiments were performed?
- (c) 4 points What are the F statistics for factors A, B, and AB (interaction)?
- (d) 2 points Which factors are significant?

## Question 2

[5 points]

An 8-run experiment was performed based on the following “sign table”. There are four factors involved: A, B, C, D.

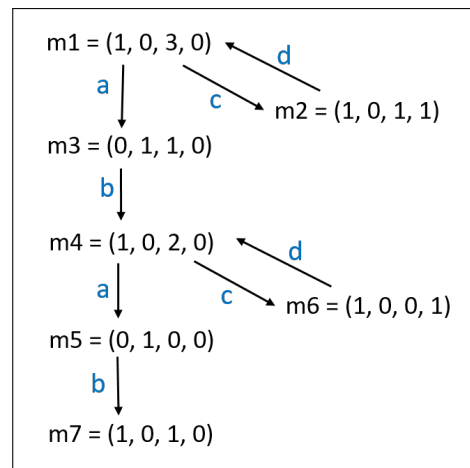
Run	A	B	C	D
1	–	–	–	–
2	+	–	–	+
3	–	+	–	+
4	+	+	–	–
5	–	–	+	+
6	+	–	+	–
7	–	+	+	–
8	+	+	+	+

- (a) 1 point What is the design here?
- (b) 1 point What is the generator function?
- (c) 3 points An alternative generator function for the experiment is  $I=ABD$ . Which one results into a better design and why?

### Question 3

[20 points]

Consider the following reachability graph for a Petri-net.



- 10 points Draw the Petri net that has produced the reachability graph.
- 3 points What is the highest liveness level (from L1-Live to L4-Live) of transition  $c$ ? Explain your answer.
- 1 point Determine the smallest  $k$  for which this Petri net is  $k$ -bounded. Briefly explain why.
- 2 points Removing which arc (between a place and a transition) will make this Petri net unbounded? Briefly justify your answer. (note, you are not allowed to change anything else except removing an arc).
- 4 points By adding only one arc, make this Petri net deadlock-free while guaranteeing that it remains bounded. (note, you are not allowed to change anything else except adding an arc).

### Question 4

[15 points]

A casino asks you to design a machine. A customer inserts a 1 Euro coin to start the game. After that, the customer pushes a red button continuously. Every time the red button is pushed, the amount can increase (or decrease) by 1 Euro with equal probability. The game continues until the customer gets to 0 Euros (in which case she loses her initial Euro) or to 3 Euros (in which case she gains 2 Euros). Let  $X_n$  denote the number of Euros in the game after  $n$  trials (each time the button is pushed represents a trial).

- 2 points Is  $X_n$  a random process? Does  $X_n$  have the Markov property? (explain your answers)
- 3 points Draw the transition diagram, and provide the one-step transition probability matrix  $P$ .
- 2 points Can you apply the identity  $\pi \times P = \pi$  to obtain the steady-state distribution? (explain your answer)
- 5 points Find the probability of reaching state 0 (0 Euros) when  $n$  is odd. That is, what is the probability of reaching state 0 after punching the red button 1, 3, 5, ... times? (The solution is an equation)
- 3 points On average, does the casino make money, lose money or break even? (substantiate your answer mathematically)

## Question 5

[15 points]

Cars arrive at a small gas station to refuel according to a Poisson process with rate 30 per hour, and have an exponential service time distribution with mean 4 minutes. Since there are four gas pumps available, four cars can refuel simultaneously, but unfortunately there is no room for cars to wait. Hence, if a car arrives when all pumps are busy, the driver leaves immediately. For each customer that is served, an average profit is made of 7 euros.

- (a) 3 points Draw the state transition diagram for the cars taking gas at the station.
- (b) 4 points Determine the probability that an arriving car is not refueled.
- (c) 2 points What is the long-run expected profit per day (consisting of eight hours)?

Some day, the manager has the opportunity to buy an adjacent parking lot, so there is room to wait for all cars that arrive when the four pumps are busy. Suppose all drivers decide to wait instead of leaving when this happens.

- (d) 5 points Determine the probability that a car has to wait.
- (e) 1 point What is the long-run expected profit per day in this case?

## Question 6

[15 points]

Consider an interactive system with a CPU and two disks (one fast and one slow). The following data was obtained by measuring the system:

Observation interval	30 minutes
Active terminals	30
Think time	12 seconds
Completed transactions	3,200
Fast disk accesses	32,000
Slow disk accesses	12,000
CPU busy	1,080 seconds
Fast disk busy	400 seconds
Slow disk busy	600 seconds

- (a) 3 points What is the response time of the system?
- (b) 4 points Determine the bottleneck component in the system.
- (c) 4 points Quantify the performance gains of upgrading the CPU to a version that is twice as fast. Consider throughput as well as response time (latency). Any remarkable finding? If so, what could be the issue?
- (d) 4 points Instead of spending money on a CPU upgrade, an alternative solution is to balance the load across the links. Compute the optimal distribution across the disks, and comment on the performance effects.