TECHNISCHE UNIVERSITEIT DELFT
Faculteit Elektrotechniek, Wiskunde en Informatica

IN4390 – Quantitative Evaluation of Embedded Systems

April 3rd, 2020, from 09:00 to 12:00

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Question:	1	2	3	4	5	6	Total
Points:	10	5	20	15	15	15	80
Score:							

- This is a closed book exam
- You may use a **simple** calculator only (i.e. graphical calculators are not permitted)
- Write your answers with a black or blue pen, not with a pencil
- **Always justify your answers, unless stated otherwise**

- The exam covers the following material:
  (a) the paper "Basic Concepts and Taxonomy of Dependable and Secure Computing" by A. Avizienis ; J.-C. Laprie ; B. Randell ; C. Landwehr
  (b) chapters 18-20,22-23 (DoE), and 30-33 (Queueing Theory) of the book "The Art of Computer Systems Performance Analysis" by R. Jain
  (c) the paper "Petri nets: Properties, analysis and applications" by T. Murata
  (d) chapters 11.2 (DTMC), and 11.3 (CTMC) of the book "Introduction to probability, statistics, and random processes" by H. Pishro-Nik
  (e) the paper "Exploring Exploration: A Tutorial Introduction to Embedded Systems Design Space Exploration" by A.D. Pimentel

Operational Laws	
Utilization law	$U = XS$
Little's law	$N = XR$
Forced-flow law	$X_k = V_k X$
Bottleneck law	$U_k = D_k X$
Operational Bounds	
Througput	$X \leq \min\left(\frac{1}{D_{max}}, \frac{N}{D+Z}\right)$
Response time	$R \geq \max\left(D, N \times D_{max} - Z\right)$
Queueing Theory M/M/1	
Utilization	$U = XS = \lambda/\mu = \rho$
Probability of n clients in the system	$P_n = \rho^n(1-\rho)$
Mean #clients in the system	$N = \rho/(1-\rho) = \lambda/(\mu-\lambda)$
Mean #clients in the queue	$N_Q = N - \rho$
Mean response time	$R = N/\lambda = 1/(\mu-\lambda)$
Mean waiting time	$W = R - S = \rho/(\mu-\lambda)$
Basic Math	
Geometric series	$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$, for $\lvert r \rvert < 1$

ANOVA Table for One Factor Experiments

Compo-nent	Sum of Squares	%Variation	DF	Mean Square	F-Comp.	F-Table
y	SSY=$\sum y_{ij}^2$		ar			
$\bar{y}_{..}$	SS0=$ar\mu^2$		1			
y-$\bar{y}_{..}$	SST=SSY-SS0	100	ar-1			
A	SSA $= r\Sigma\ \alpha_i^2$	$100\left(\frac{\text{SSA}}{\text{SST}}\right)$	a-1	MSA $= \frac{\text{SSA}}{a-1}$	$\frac{\text{MSA}}{\text{MSE}}$	$F_{[1-\alpha; a-1, a(r-1)]}$
e	SSE=SST- SSA	$100\left(\frac{\text{SSE}}{\text{SST}}\right)$	$a(r-1)$	MSE $= \frac{\text{SSE}}{a(r-1)}$		

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Question 1 [10 points]

A two-factor ANOVA table is obtained through a factorial design.

Two-way ANOVA: y versus A, B

Source	DF	SS	MS	F	P
A	1	____	0.002	____	0.998
B	_	180.375	____	____	0.093
Interaction	3	8.479	____	____	0.932
Error	8	____	____		
Total	15	347.653			

(a) $\boxed{\text{2 points}}$ How many levels were used for factor B?

Solution: DFAB = DFA * DFB, so DFB = DFAB / DFA = 3 / 1 = 3. Then factor B had 4 levels.

(b) $\boxed{\text{2 points}}$ How many replicates of the experiments were performed?

Solution: DFE = ab(r-1), so r = $8/(2 \times 4) + 1 = 2$ replications.

(c) $\boxed{\text{4 points}}$ What are the F statistics for factors A, B, and AB (interaction)?

Solution: We need to compute the MSE, thus we need to find SSE = SST - SSA - SSB - SSAB. SSA is unknown, but follows directly from the rule MSA = SSA/DFA. SSA = 0.002, then SSE = 347.653 - 0.002 - 180.375 - 8.479 = 158.797, which leads to MSE = SSE/DFE = 158.797 / 8 = 19.85

FA = MSA/MSE = 0.002 / 19.85 = 0.0001

MSB = SSB/DFB = 180.358 / 3 = 60.119, so FB = MSB/MSE = 60.119 / 19.85 = 3.03

MSAB = SSAB/DFAB = 8.479 / 3 = 2.826, so FAB = MSAB/MSE = 2.826 / 19.85 = 0.142

(d) $\boxed{\text{2 points}}$ Which factors are significant?

Solution: Since the p-values are given, we can simply check if those pass the bar for significance. That is, check if p-value < significane level α. Typical values for α are in the 5-10% range. That renders factor B moderately significant (0.093 < 0.10), while factor A (p = 0.998) and the interaction (p = 0.932) are definitively **not** significant.

Question 2 [5 points]

An 8-run experiment was performed based on the following "sign table". There are four factors involved: A, B, C, D.

Run	A	B	C	D
1	−	−	−	−
2	+	−	−	+
3	−	+	−	+
4	+	+	−	−
5	−	−	+	+
6	+	−	+	−
7	−	+	+	−
8	+	+	+	+

(a) $\boxed{\text{1 point}}$ What is the design here?

Solution: One half factorial design, 2^{4-1}

(b) $\boxed{\text{1 point}}$ What is the generator function?

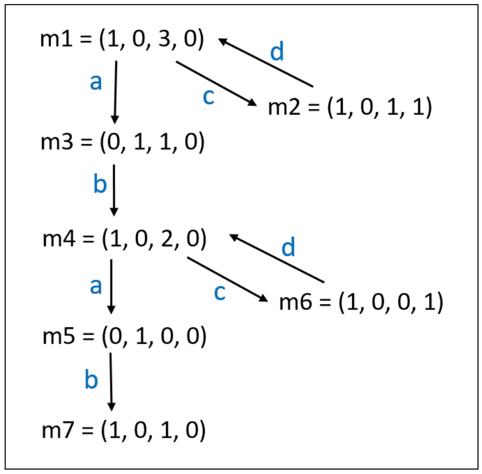
Solution: I=ABCD (or any of its aliases)

(c) $\boxed{\text{3 points}}$ An alternative generator function for the experiment is I=ABD. Which one results into a better design and why?

Solution: We need to look at the resolutions as the higher the resolution, the better. Generator I=ABCD has resolution IV, while generator I=ABD has resolution III, hence, generator I=ABCD is the better design.

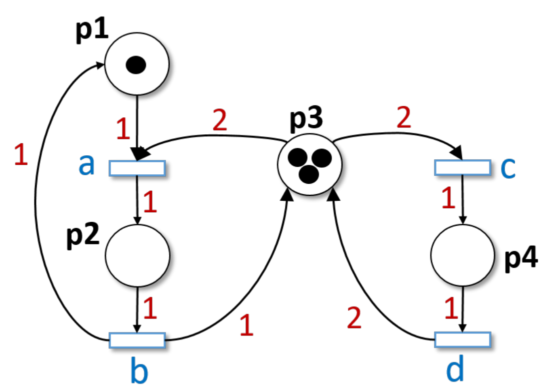
Question 3 [20 points]

Consider the following reachability graph for a Petri-net.



(a) 10 points Draw the Petri net that has produced the reachability graph.

Solution:



(b) 3 points What is the highest liveness level (from L1-Live to L4-Live) of transition c? Explain your answer.

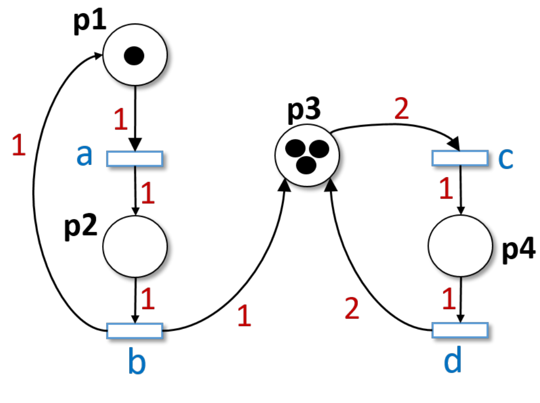
Solution: c is L3-Live because it can fire infinitely often between markings m1 and m2 or between markings m4 and m6. It is not L4-Live because it is NOT L1-live for every marking that is reachable from m1. For example, transition c does not fire even once in the following sequence of markings: $\langle m1, m3, m4, m5, m7 \rangle$.

(c) 1 point Determine the smallest k for which this Petri net is k-bounded. Briefly explain why.

Solution: It is 3-bounded, i.e., $k = 3$, because in m1, there are 3 tokens in P3 and that is the largest number of tokens that can gather in one place in all of the reachable markings.

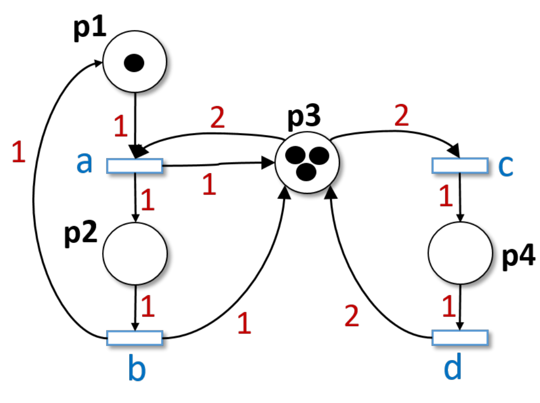
(d) 2 points Removing which arc (between a place and a transition) will make this Petri net unbounded? Briefly justify your answer. (note, you are not allowed to change anything else except removing an arc).

Solution: By removing the arc between P3 and transition a, there is no need to remove tokens from P3 in order to fire transition a. Hence, the token in P1 can move to P2 and go back to P1 without any restriction. However, each time it passes through transition b, one token will be added to P3. Since there are unlimited number of time that b can fire, there will be unlimited number of tokens that can be stored in P3.



(e) $\boxed{\text{4 points}}$ By adding only one arc, make this Petri net deadlock-free while guaranteeing that it remains bounded. (note, you are not allowed to change anything else except adding an arc).

Solution: By adding an arc with label 1 between transition a and place P3 as shown in the figure below, the Petri net becomes deadlock free because each time two tokens are removed from P3 to fire a, immediately, one of them will be put back to P3. The other one will be back to P3 after firing b, hence, when P1 gets back its token, P3 has also received its two tokens.



Question 4 [15 points]

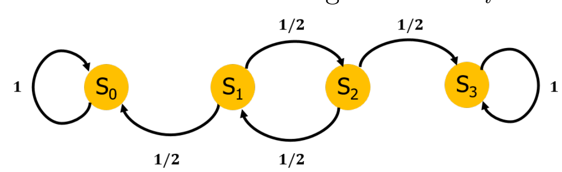
A casino asks you to design a machine. A customer inserts a 1 Euro coin to start the game. After that, the customer pushes a red button continuously. Every time the red button is pushed, the amount can increase (or decrease) by 1 Euro with equal probability. The game continues until the customer gets to 0 Euros (in which case she loses her initial Euro) or to 3 Euros (in which case she gains 2 Euros). Let X_n denote the number of Euros in the game after n trials (each time the button is pushed represents a trial).

(a) $\boxed{\text{2 points}}$ Is X_n a random process? Does X_n have the Markov property? (explain your answers)

Solution: Yes, it is a random process because every instance of X_n is a random variable (+1 or -1). Yes, the random process has the Markov property because the future state (amount of money) depends only on the present state.

(b) $\boxed{\text{3 points}}$ Draw the transition diagram, and provide the one-step transition probability matrix P.

Solution: Transition diagram with S_i denoting that there are i euros at stake.



$$P = \begin{pmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.5 & 0.0 & 0.5 & 0.0 \\ 0.0 & 0.5 & 0.0 & 0.5 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix}$$

(c) $\boxed{\text{2 points}}$ Can you apply the identity $\pi \times P = \pi$ to obtain the steady-state distribution? (explain your answer)

Solution: No, because the chain is reducible. The system cannot reach other states once it gets to state S_0 or S_3.

(d) $\boxed{\text{5 points}}$ Find the probability of reaching state 0 (0 Euros) when n is odd. That is, what is the probability of reaching state 0 after punching the red button 1, 3, 5, ... times? (The solution is an equation)

Solution: One option is to calculate P^n, for different values of n until one can spot a trend. But that is cumbersome, so let's consider how the probability mass evolves with subsequent trials by creating a table.

At n = 0, all the probability mass is in state 1. After that, the probability flows at every step according to the transition diagram as follows:

trial	P_0	P_1	P_2	P_3
n=0	0	1	0	0
n=1	1/2	0	1/2	0
n=2	1/2	1/4	0	1/4
n=3	1/2 + 1/8	0	1/8	1/4
n=4	1/2 + 1/8	1/16	0	1/4 + 1/16
n=5	1/2 + 1/8 + 1/32	0	1/32	1/4 + 1/16

For the final equation one can identify the pattern: $P_0^n = \sum_{i=1,odd}^{n} \frac{1}{2^i}$

(e) $\boxed{\text{3 points}}$ On average, does the casino make money, lose money or break even? (substantiate your answer mathematically)

Solution: With the equation obtained in (d) we can obtain the sum when n goes to infinity

X = 1/2 + 1/8 + 1/32 + ...

Now this looks like a geometric series with halve the items missing; the items that we obtain by "shifting" X with a factor of 2

2X = 1 + 1/4 + 1/16 + ...

Which allows us to write

geom(1/2) = 2X + X = 2, hence, X = 2/3

Now it follows that the casino **breaks even** because:

* with probability 2/3, the customer will lose the Euro it deposited at the beginning. On expectation, the customer loses 2/3 of a Euro

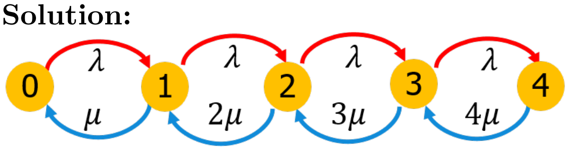
* with probability 1/3, the customer will win 2 Euros (the game ends when the game reaches 3 Euros). On expectation, the customer wins 2/3 of a Euro (1/3 * 2 Euros).

Question 5 [15 points]

Cars arrive at a small gas station to refuel according to a Poisson process with rate 30 per hour, and have an exponential service time distribution with mean 4 minutes. Since there are four gas pumps available, four cars can refuel simultaneously, but unfortunately there is no room for cars to wait. Hence, if a car arrives when all pumps are busy, the driver leaves immediately. For each customer that is served, an average profit is made of 7 euros.

(a) 3 points Draw the state transition diagram for the cars taking gas at the station.

Solution:



(b) 4 points Determine the probability that an arriving car is not refueled.

Solution: $\lambda = 30/60 = 1/2$ and $\mu = 1/4$; let $\tau = \lambda/\mu = 2$

$$\begin{array}{ll} \lambda P_0 = \mu P_1 & P_1 = \tau P_0 = 2P_0 \\ \lambda P_1 = 2\mu P_2 & P_2 = \frac{1}{2}\tau P_1 = 2P_0 \\ \lambda P_2 = 3\mu P_3 & P_3 = \frac{1}{3}\tau P_2 = \frac{4}{3}P_0 \\ \lambda P_3 = 4\mu P_4 & P_4 = \frac{1}{4}\tau P_3 = \frac{2}{3}P_0 \end{array}$$

Factoring in that $\sum_{n=0}^{4} P_n = 1$ yields the following expression:

$P_0 = \frac{1}{1+2+2+4/3+2/3} = 3/21 = 1/7$

The probability of leaving immediately is entering the gas station when all four pumps are occupied, which equals $P_4 = \frac{2}{3}P_0 = 2/21$

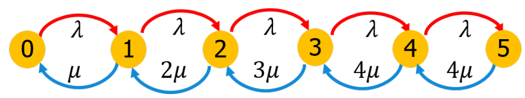
(c) 2 points What is the long-run expected profit per day (consisting of eight hours)?

Solution: Profit = all cars taking gas $\times$ 7 = $8\,hrs \times 30\,cars/hr \times (1 - P_4) \times 7\,euro/car$ $= 8 \times 30 \times 19/21 \times 7 = 80 \times 19 = 1520$ euro.

Some day, the manager has the opportunity to buy an adjacent parking lot, so there is room to wait for all cars that arrive when the four pumps are busy. Suppose all drivers decide to wait instead of leaving when this happens.

(d) $\boxed{\text{5 points}}$ Determine the probability that a car has to wait.

Solution: The number of states now grows to infinity (an M/M/4 queue):



...

The equations need to be extended, leading to a geometric series starting at P_3:

$$\begin{array}{ll} \ldots & \text{as before} \\ \lambda P_3 = 4\mu P_4 & P_4 = \frac{1}{4}\tau P_3 = \frac{1}{2}P_3 \\ \lambda P_4 = 4\mu P_5 & P_5 = \frac{1}{4}\tau P_4 = (\frac{1}{2})^2 P_3 \\ \lambda P_5 = 4\mu P_6 & P_6 = \frac{1}{4}\tau P_5 = (\frac{1}{2})^3 P_3 \\ \ldots & \end{array}$$

Using the closed form for the geometric series we find:

$\sum_{i=3}^{\infty} P_i = \sum_{i=0}^{\infty} (\frac{1}{2})^i \times P_3 = geom(1/2) \times P_3 = 2P_3$

Now we can solve P_0:

$P_0 = \frac{1}{1+2+2+2\times 4/3} = 3/23$

Drivers have to wait when ≥ 4 cars are taking gas, which equals to

$\sum_{i=4}^{\infty} P_i = (\sum_{i=3}^{\infty} P_i) - P_3 = 2P_3 - P_3 = P_3 = \frac{4}{3}P_0 = 4/3 \times 3/23 = 4/23$

(e) $\boxed{\text{1 point}}$ What is the long-run expected profit per day in this case?

Solution: Since all cars will take gas, the profit simply equals $8 \times 30 \times 7 = 1680$ euros.

Question 6 [15 points]

Consider an interactive system with a CPU and two disks (one fast and one slow). The following data was obtained by measuring the system:

Observation interval	30 minutes
Active terminals	30
Think time	12 seconds
Completed transactions	3,200
Fast disk accesses	32,000
Slow disk accesses	12,000
CPU busy	1,080 seconds
Fast disk busy	400 seconds
Slow disk busy	600 seconds

(a) $\boxed{\text{3 points}}$ What is the response time of the system?

Solution: We can apply Little's law (for closed systems): $N = X(R+Z)$. The throughput of the system is $X = C/T = 3200/1800 = 16/9 = 1.78$ trans/sec. The response time (per transaction) is then $R = N/X - Z = 30 \times 9/16 - 12 = 4.875$ sec.

(b) $\boxed{\text{4 points}}$ Determine the bottleneck component in the system.

Solution:

- $U_{CPU} = B_{CPU}/T = 1080/1800 = 60\%$
- $U_{fast} = B_{fast}/T = 400/1800 = 22.2\%$
- $U_{slow} = B_{slow}/T = 600/1800 = 33.3\%$

Thus the CPU is the bottleneck.

(c) $\boxed{\text{4 points}}$ Quantify the performance gains of upgrading the CPU to a version that is twice as fast. Consider throughput as well as response time (latency). Any remarkable finding? If so, what could be the issue?

Solution: The CPU utilization then drops to 30%, which causes the slow disk to become the bottleneck. System throughput can then increase upto a factor of 60 / 33.3 = 1.8 Thus the throughput rises to X = 1.8 × 16/9 = 16/5 = 3.2 trans/sec.

Recomputing the system response leads to R = $R = N/X - Z = 30 \times 5/16 - 12 = -2.625$ sec. Wow, a negative response time! The problem is that the number of active terminals can only produce a maximum load of 30 × 1/12 = 2.5 trans/sec, and thus the think time has become the bottleneck in the system.

(d) $\boxed{\text{4 points}}$ Instead of spending money on a CPU upgrade, an alternative solution is to balance the load across the links. Compute the optimal distribution across the disks, and comment on the performance effects.

Solution: We start with computing the access times of the two disks.

- $S_{fast} = B_{fast}/C_{fast} = 400/32000 = 1/80 = 12.5$ ms.
- $S_{slow} = B_{slow}/C_{slow} = 600/12000 = 1/20 = 50$ ms.

Now we want to balance the accesses over the disks to equalize the times spent at each disk:

- $\mathrm{S}_{slow} \times \mathrm{C}_{slow} = \mathrm{S}_{fast} \times \mathrm{C}_{fast}$
- $\mathrm{C}_{slow} + \mathrm{C}_{fast} = 12{,}000 + 32{,}000 = 44{,}000$

Solving the above set of equations leads to $C_{fast} = 4 \times C_{slow} = 35,200$ equalizing the utilization at $U_{slow} = U_{fast} = 35200/32000 \times 22.2 = 24.4\%$

The load balancing is effective in itself, yet without upgrading the CPU, the system performance will not be (notably) impacted (the CPU remains the bottleneck with a utilization of 60%).