CS4140
Embedded Systems Laboratory

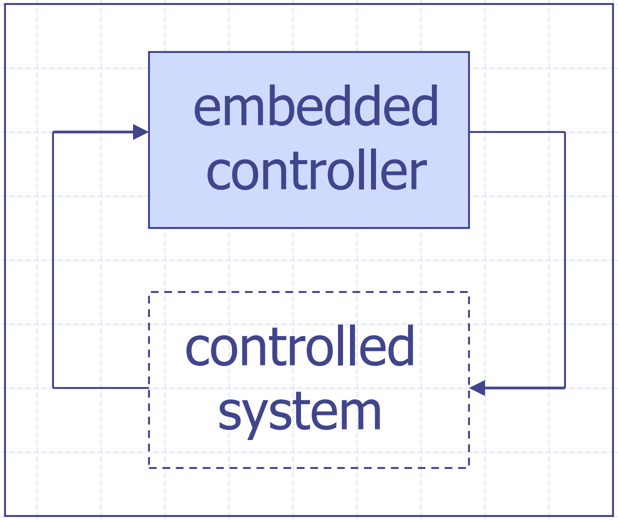
Introduction to Control Theory

Why Control Theory?

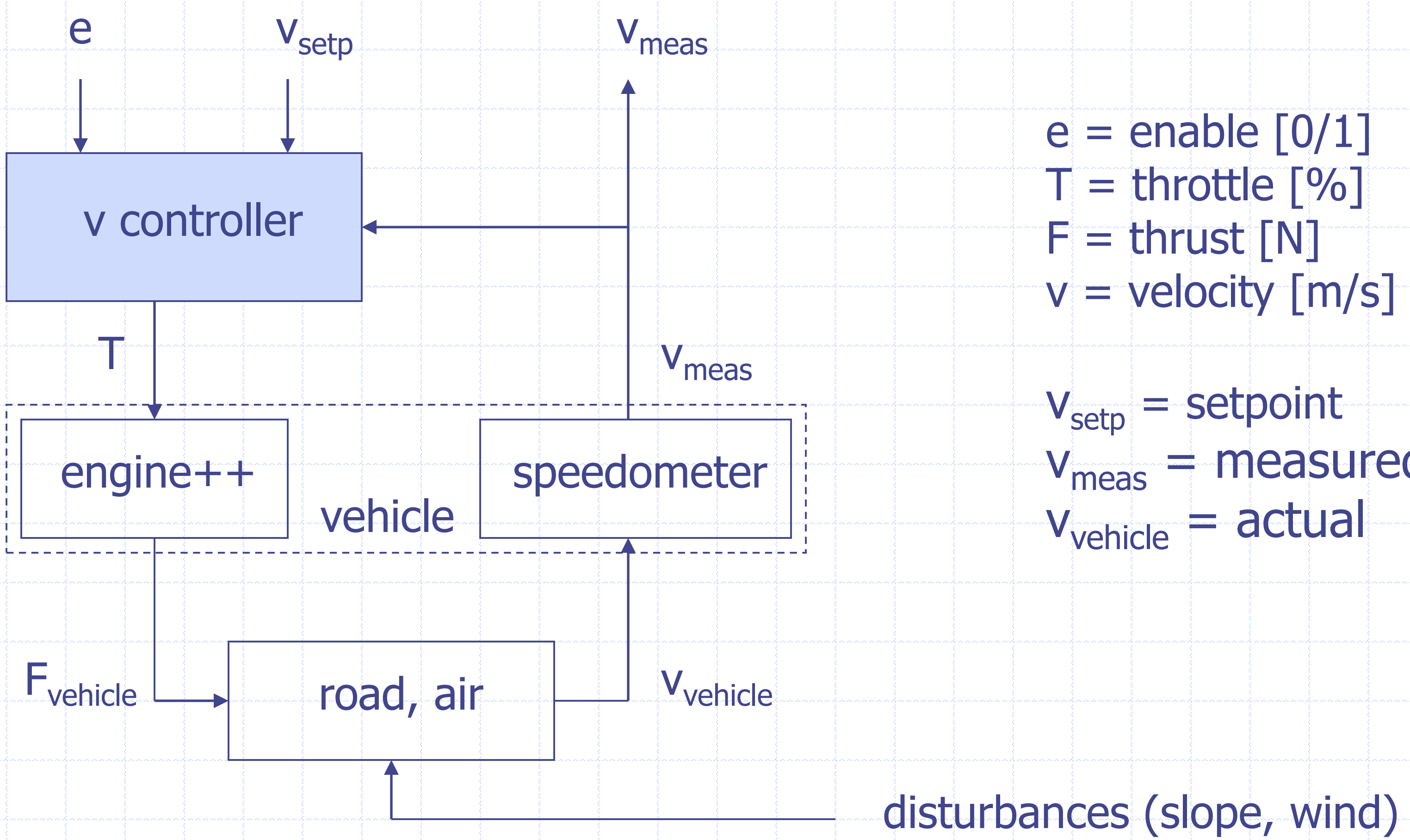
- Embedded systems integrated with appl'n
- Multi-disciplinary training required:
  - Physics engineering
  - Electronics engineering
  - Mechanical engineering
  - Control engineering
  - …
  - And, of course,
  - Computer science & engineering

Control is Everywhere

- Automotive
- Aerospace
- Plant Control
- Climate Control
- Health Care
- Copiers, Wafer Scanners
- Model Quad Rotors ...



Cruise Control



e = enable [0/1]
T = throttle [%]
F = thrust [N]
v = velocity [m/s]

v_{setp} = setpoint
v_{meas} = measured
$v_{vehicle}$ = actual

Objectives of this Crash Course

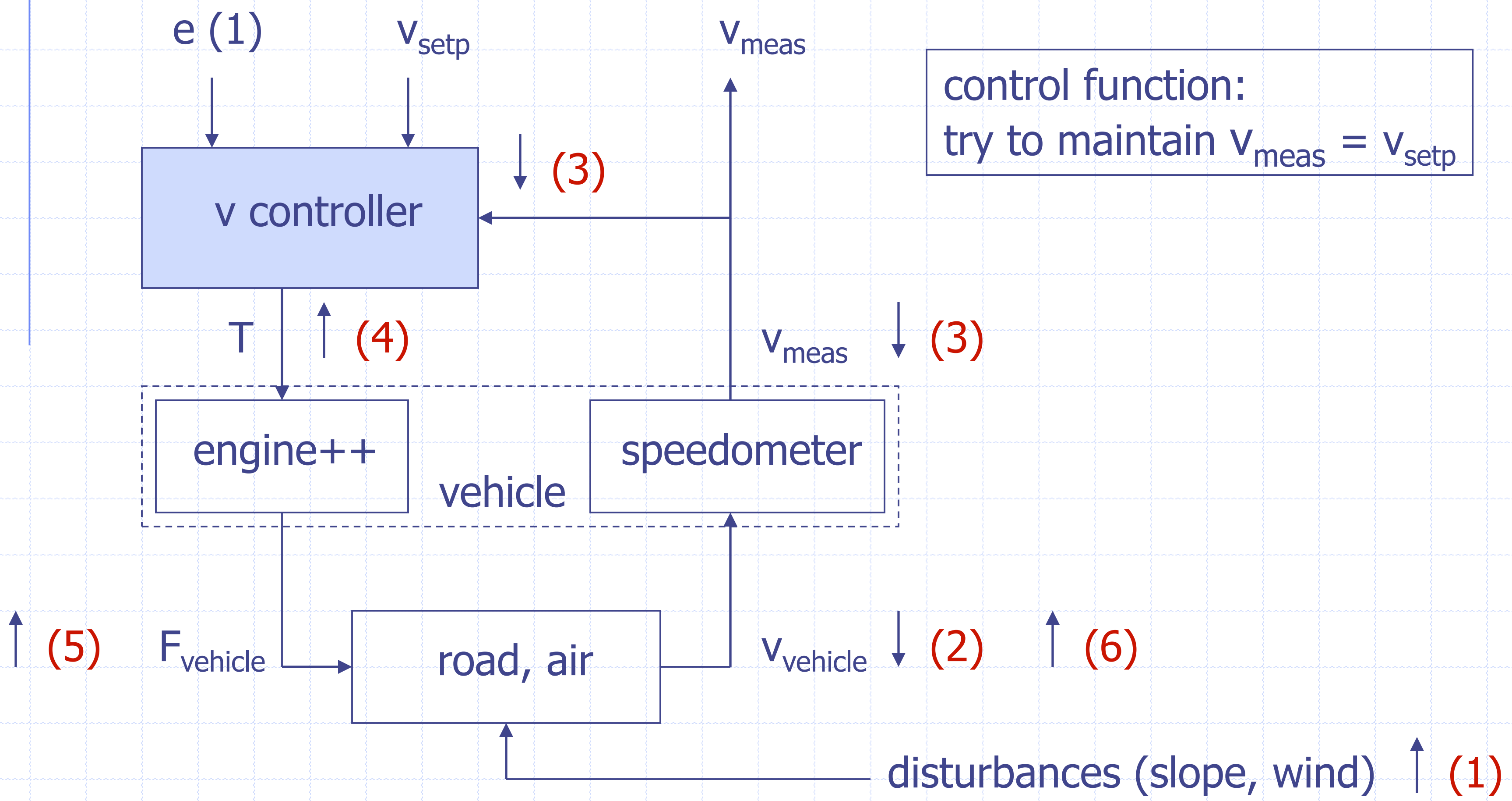
- Appreciate the benefits of control
- Understand basic control principles
- Communicate with control engineers

- Get you up to speed to do the QR control

Part I: Feedback Control

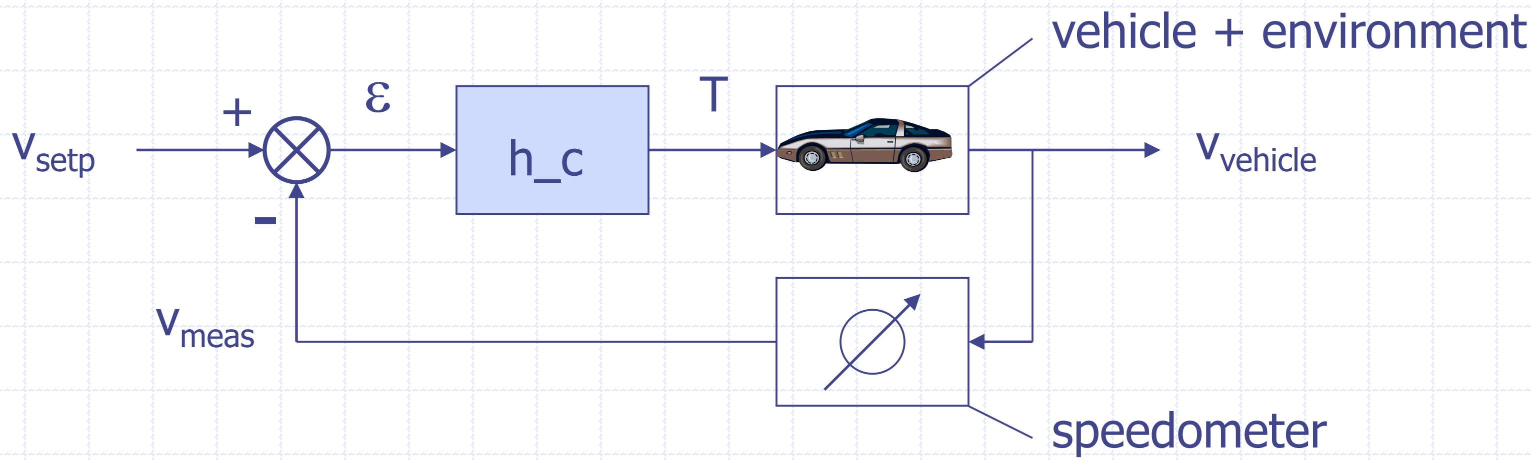
- What is Control
- The Feedback Loop
- Proportional Feedback

Velocity Control



control function:
try to maintain $v_{meas} = v_{setp}$

Feedback Control Loop

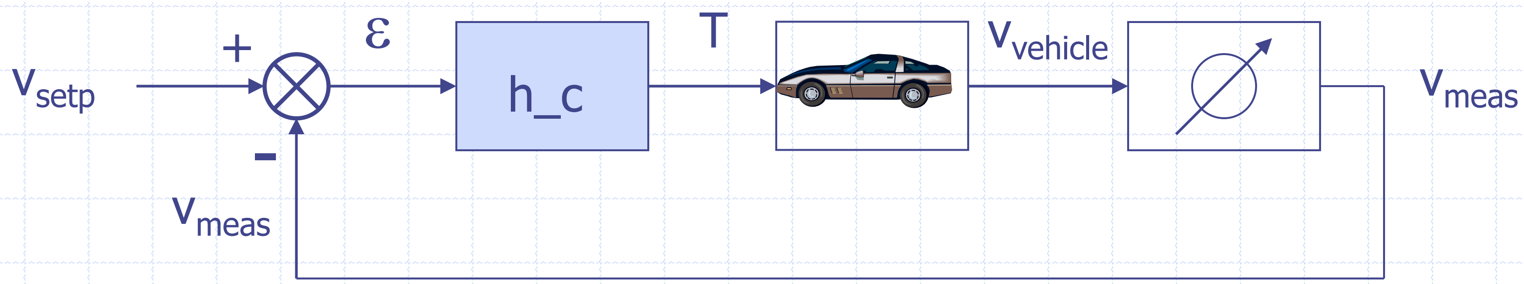


h_c controller

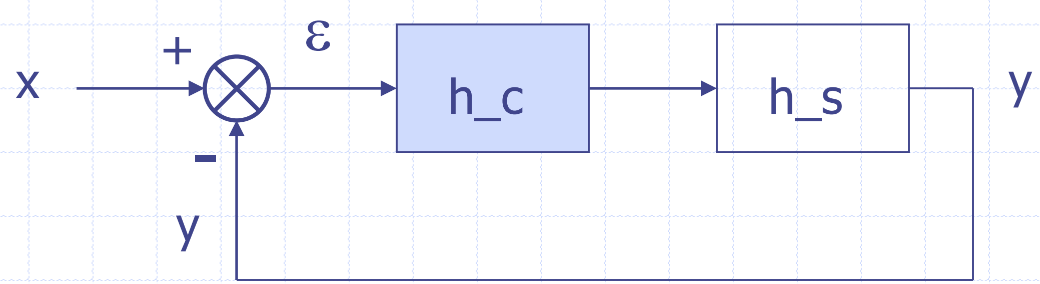
controller function T=h_c(ε): adjust T such that ε -> 0

control theory: how to determine function h_c

Standard Loop Format



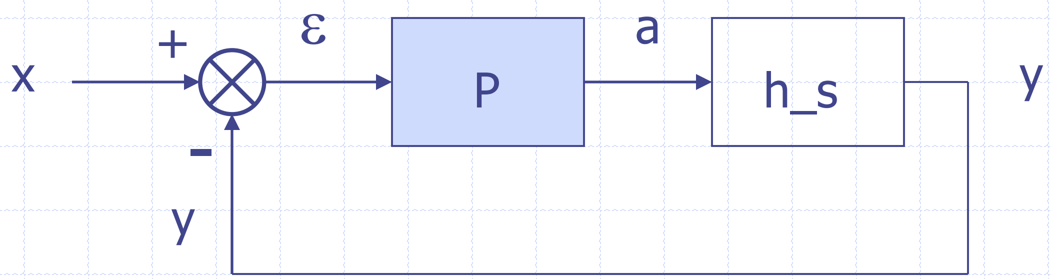
<u>standard form</u>: control h_s through h_c such that y = x



h_c = h_controller
h_s = h_system

Proportional Control

Let h_c(ε) = P ε

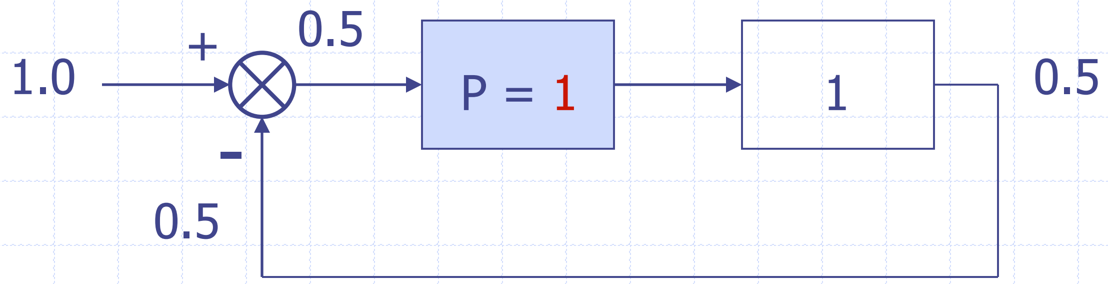


(Steady-state) Analysis:

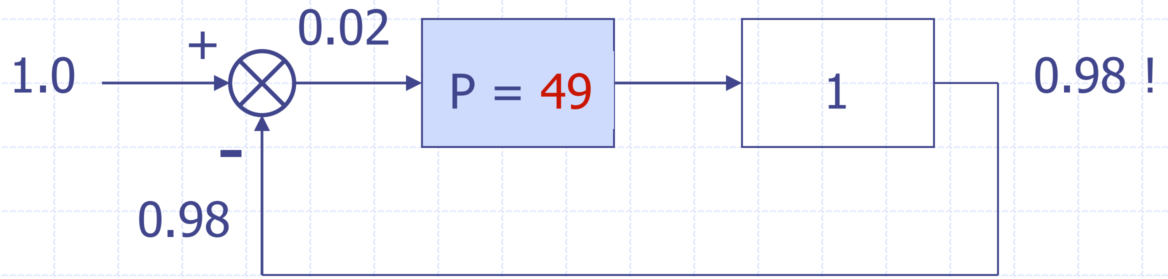
Let h_s(a) = c a (i.e. linear system)
Then y = c P (x-y) => y = (c P/(c P+1)) x

Effect of Loop Gain

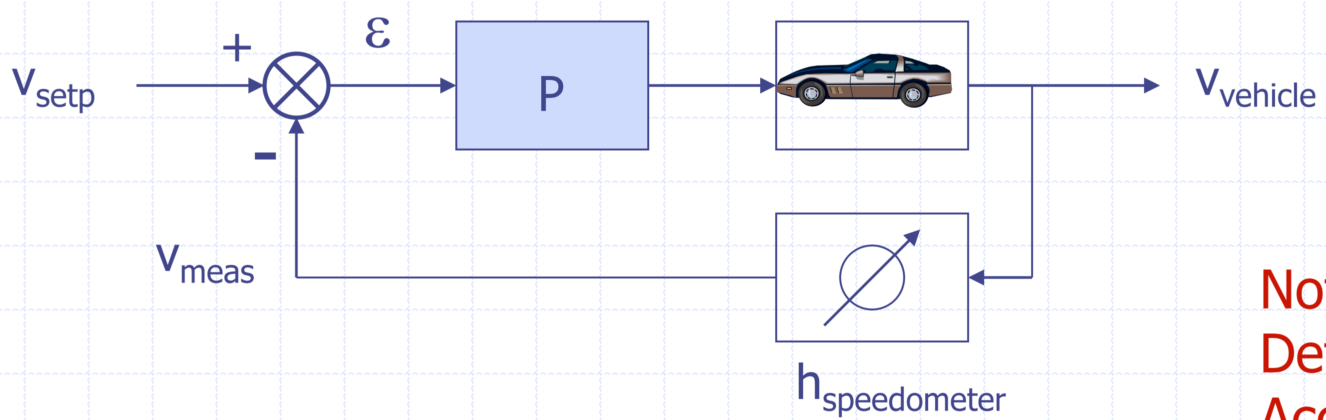
$y = P/(P+1)\ x$



Loop gain: the larger, the better ($y \approx x$)



Example: Velocity Control



Note: Sensor Determines Accuracy

Analysis:

$v_{meas} = h_{speedometer}(v_{vehicle})$

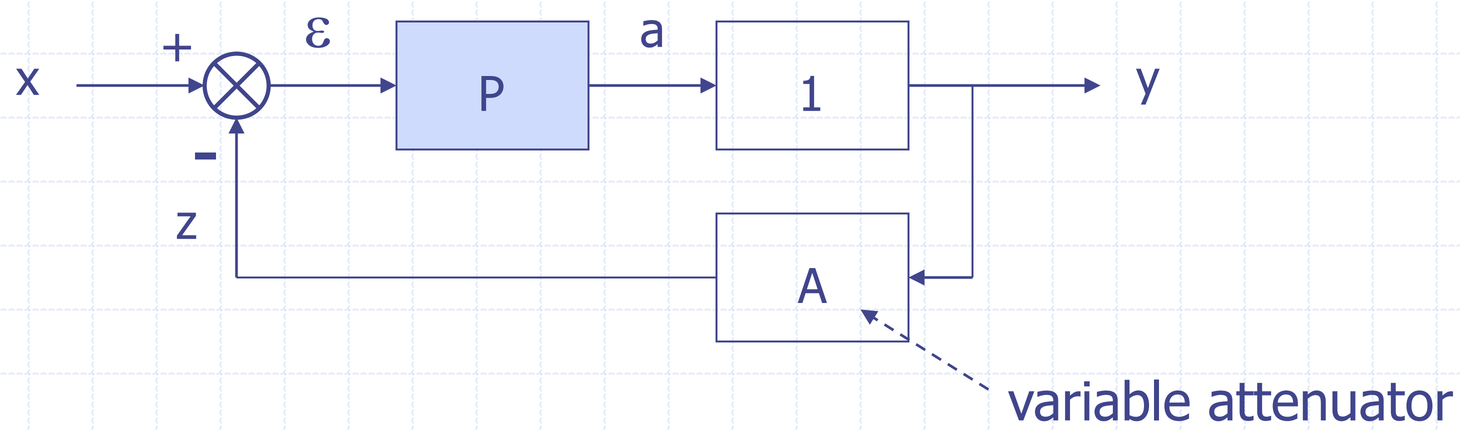
If $P \gg 1$ then $v_{meas} \approx v_{setp}$

Consequently, $v_{vehicle} \approx h_{speedometer}^{-1}(v_{setp})$

Ideally, $h_{speedometer}(x) = x$

Result: $v_{vehicle} \approx v_{setp}$

Example: Variable Amplifier



Analysis:

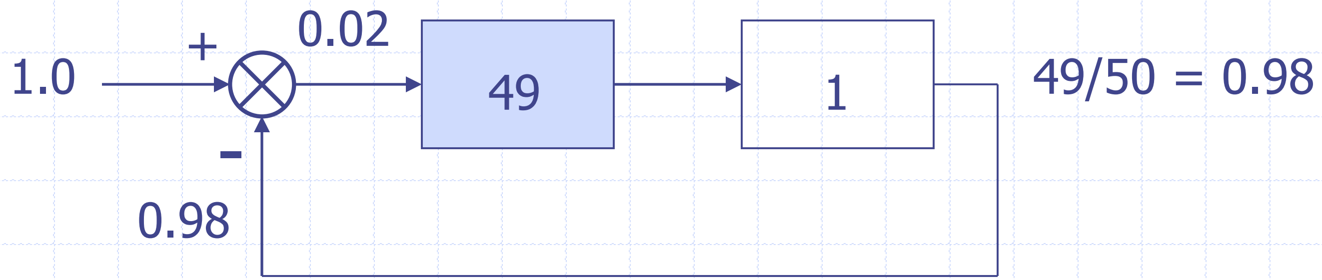
If P A » 1 (i.e. sufficient loop gain) then z ≈ x
Hence y ≈ (1/A) x (e.g. A = 0.001 => 1000 x amp)

Part II: Blessings of Feedback

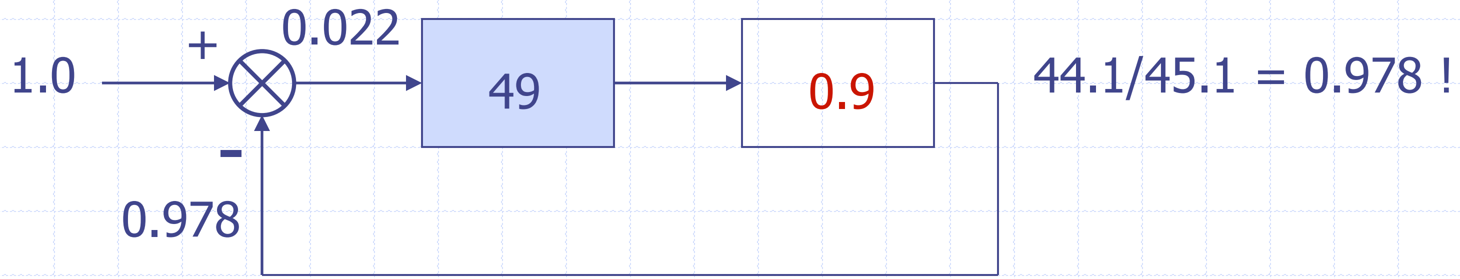
- High Loop Gain: More Robustness
- High Loop Gain: More Linearity
- High Loop Gain: More Speed

More Robustness

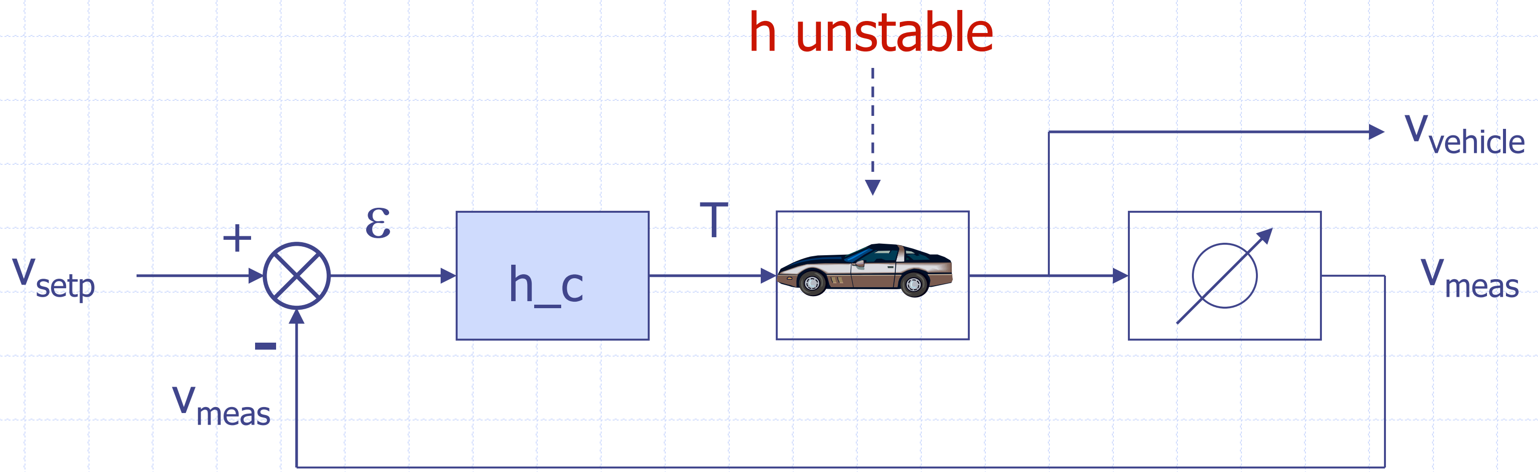
- Suppose h_s varies with time



- 10% change in h_s: only 10%/50 = 0.2% change in y



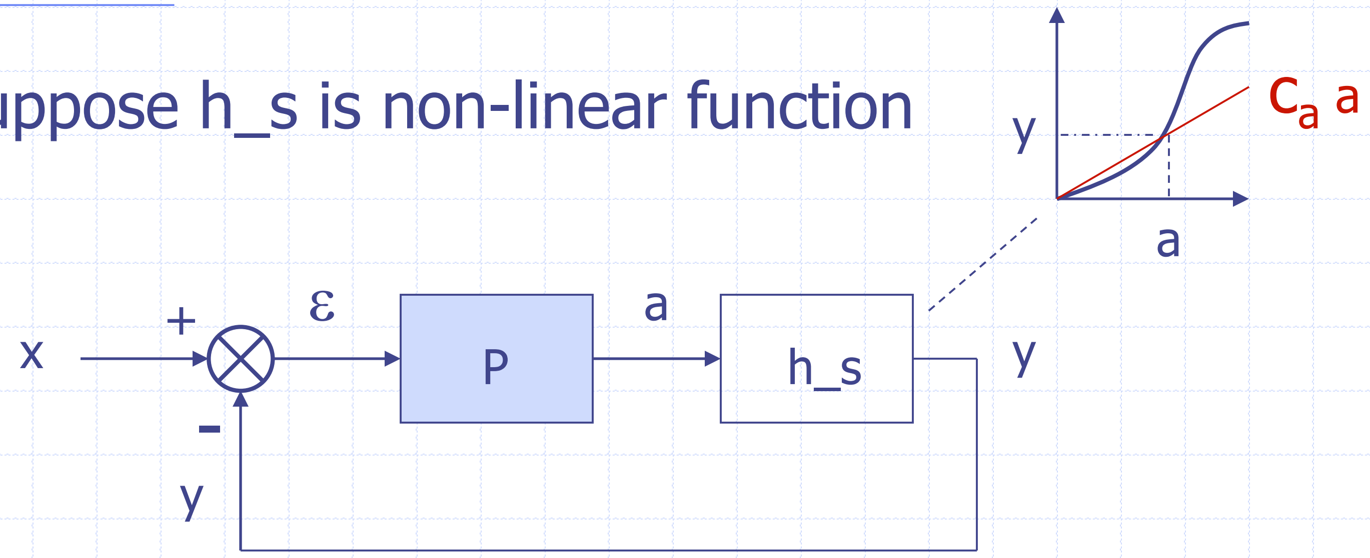
Example: Velocity Control



For sufficiently high loop gain: v_{meas} stable ($\approx v_{setp}$),
Hence $v_{vehicle} \approx h_{speedometer}^{-1}(v_{setp})$, which is stable

More Linearity

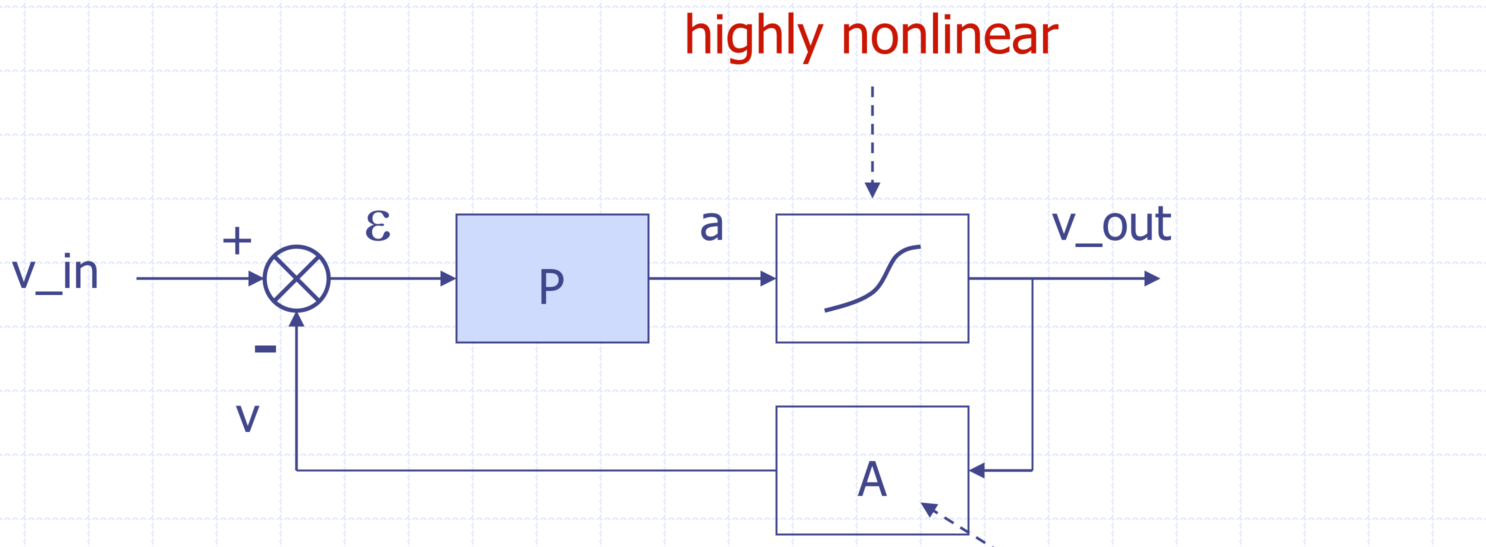
Suppose h_s is non-linear function



Analysis:

Let h_s(a) = c_a a => y = (c_a P/(c_a P+1)) x
If c_a P » 1 then y ≈ x => y is <u>linear</u> with x

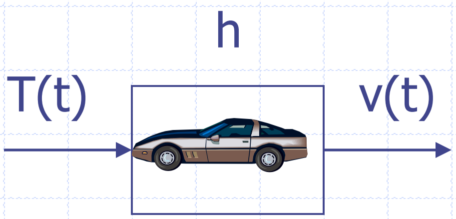
Example: Audio Amp



Analysis:

If c_a P A » 1 then v ≈ v_in
Hence v_out ≈ 1/A v_in (so linear gain: 1/A)

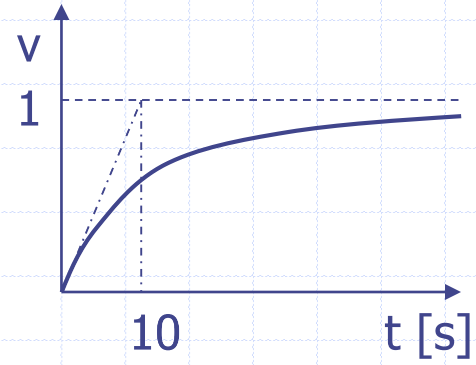
More Speed



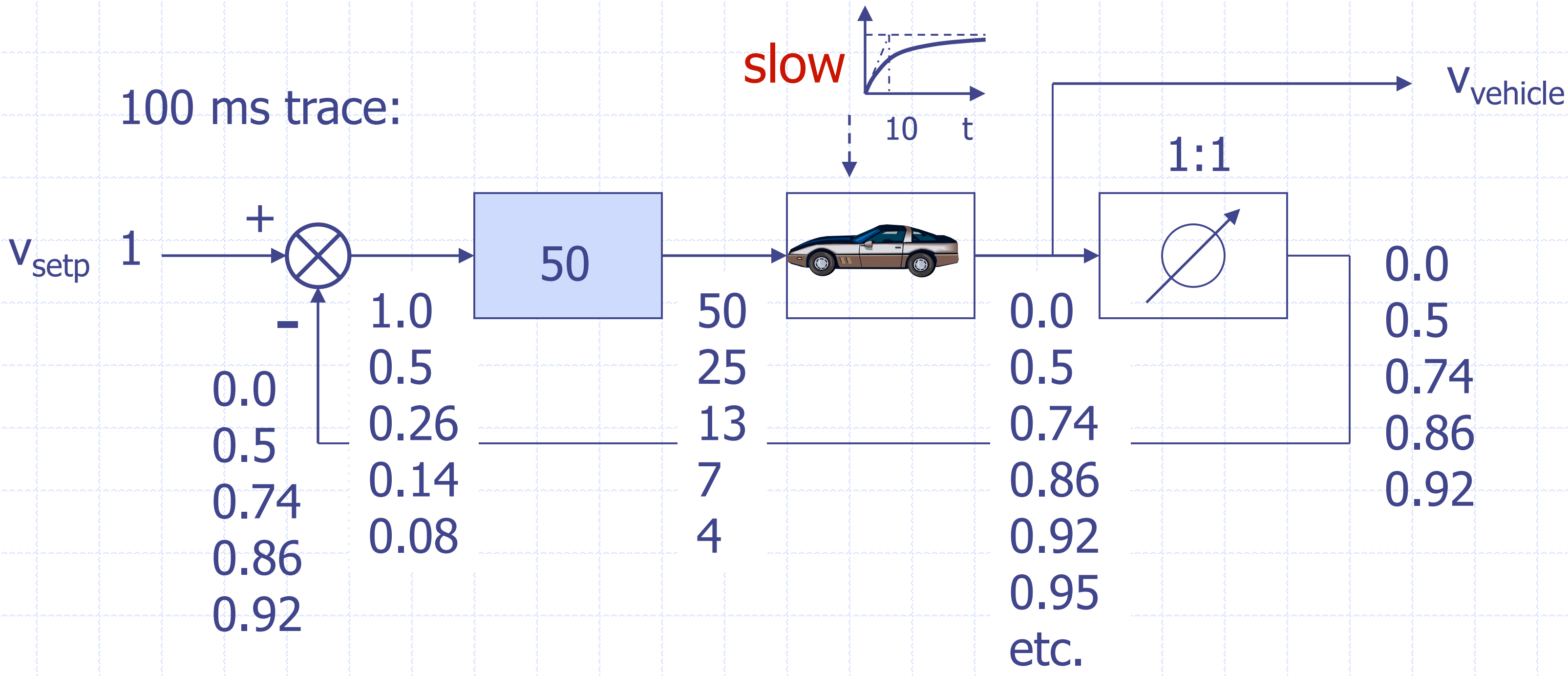
T and v are typically time-varying signals (function of t). transfer function (h) is not just a proportional gain function but a <u>first-order transfer function</u>:

Vehicle response (slow):
$10(dv(t)/dt) + v(t) = T(t)$

Let $T(t) = 1 \Rightarrow$
$v(t) = 1 - e^{-t/10}$



Example: Velocity Control



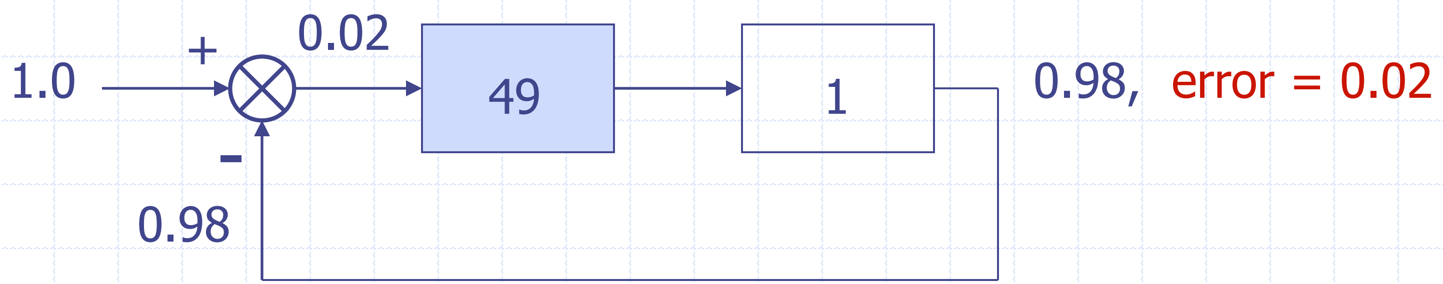
In 2 steps of 100 ms same level (0.74) as 10 s w/o feedback
Performance of vehicle has effectively increased ~50 times!

Part III: Harnessing Feedback

- Instability Problem
- Classical Control Theory

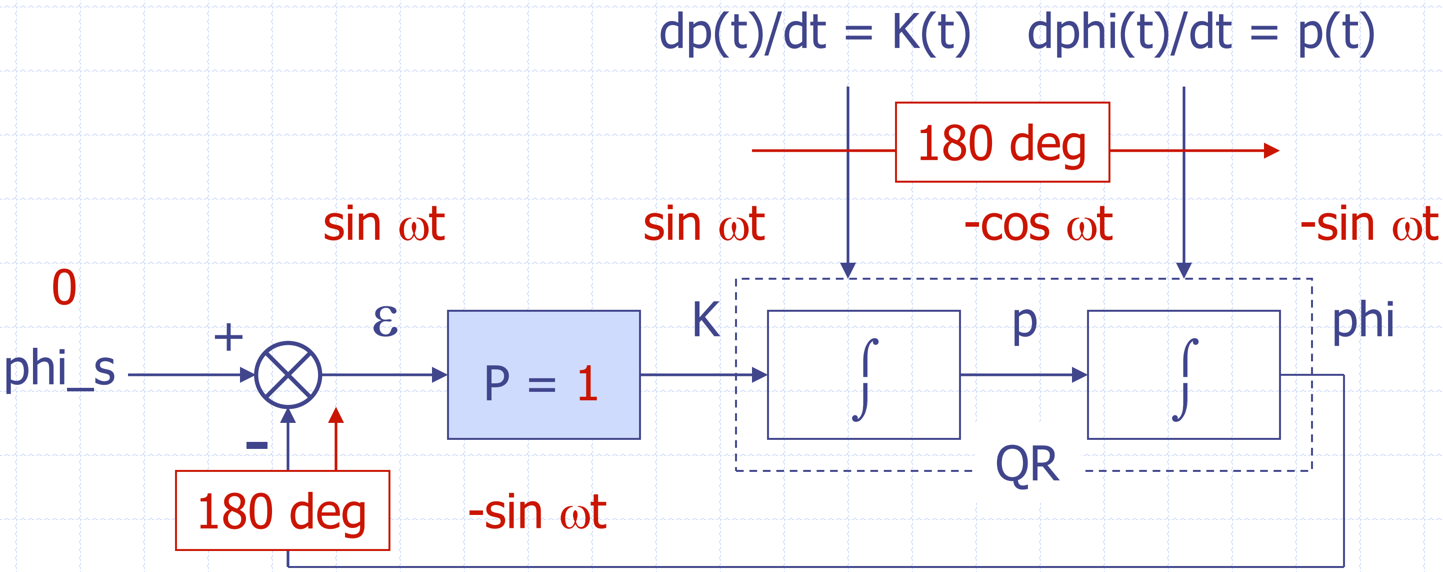
Loop Gain Limitations

Analysis: y = P/(P+1) x



Problem:
P should be infinite for control error to become zero
In practice however, loop gain must be limited for *stability*

Example 1: Integrator Systems

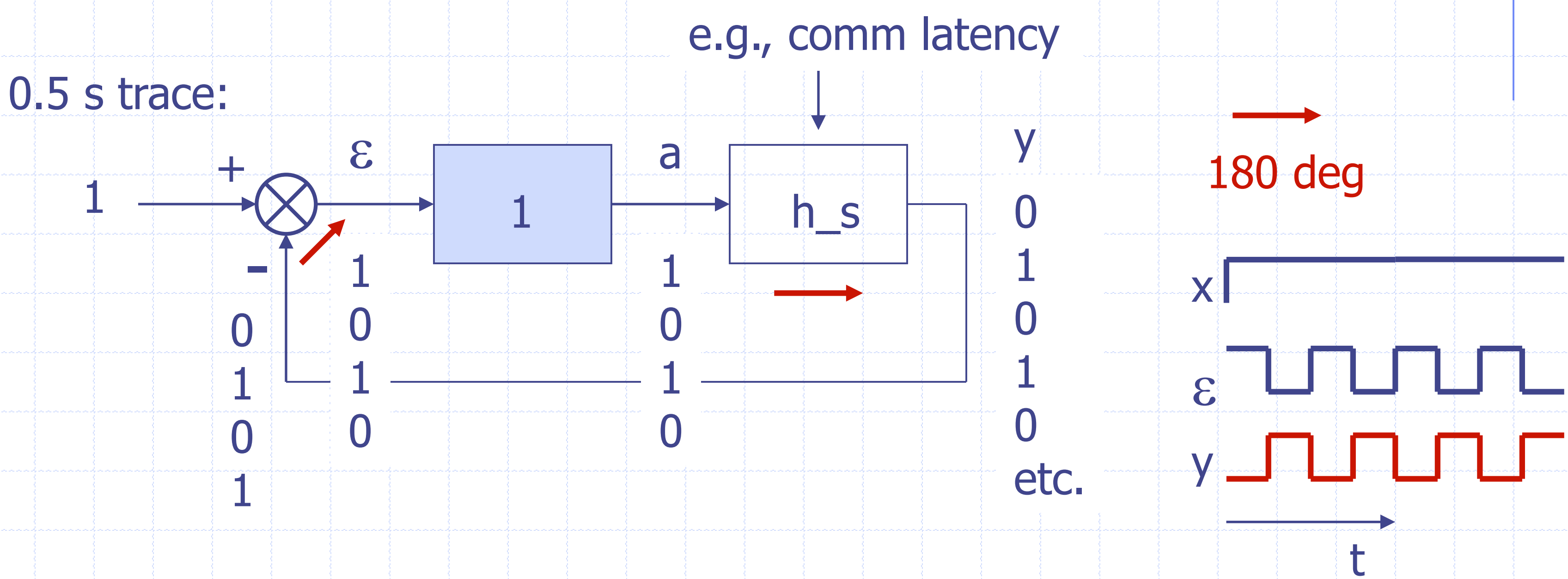


P ≥ 1: instability!

Cause: each integration adds 90 deg phase lag

So 2 integrators use up all 180 deg budget!

Example 2: Time Latency



Let h_s: y(t) = a(t-0.5)   (i.e., 0.5s delay)

Phase lag of 180 deg at 1 Hz causes instability

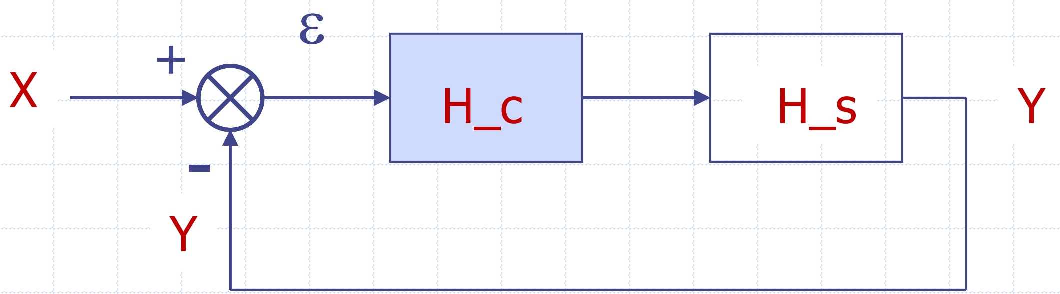
Phase Lag: examples

h_s

- Integration (90 deg):
  - speed -> position, flow -> volume
- First-order system (up to 90 deg):
  - lamp, heating, car velocity, ...
- N-th order system (up to N*90 deg):
  - compositions of 1st-order systems, missiles
- Delay systems (unlimited):
  - humans, computers, sample times, cables, air

Need control theory to analyze, e.g., control stability

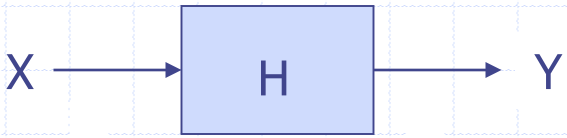
Classical Control Theory



Describe x(t), y(t), h_c(t), h_s(t) in terms of their Laplace transforms X(s), Y(s), H_c(s), H_s(s), respectively

$$L[f(t)] = F(s) = \int_0^\infty f(t) e^{-st} dt$$

Classical Control Theory



For <u>linear</u> system h it holds Y(s) = H(s) · X(s)
(i.e. composition in time domain reduces to multiplication in the Laplace domain). This allows for easy analysis.

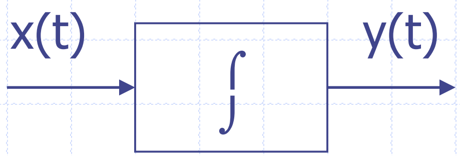
Laplace cheat sheet

- $L[a] = a/s$
- $L[a\,t] = a / s^2$
- $L[a\,f + b\,g] = a\,L[f] + b\,L[g]$
  $= a\,F(s) + b\,G(s)$
- $L[f\,'] = s\,L[f] - f(0)$
  $= s\,F(s) - f(0)$

Example: Rate Control (1)

Laplace cheat sheet

- $L[a] = a/s$
- $L[a\,t] = a / s^2$
- $L[f+g] = F(s) + G(s)$
- $L[f'] = s\,F(s) - f(0)$
- $L[g(f)] = F(s)\,G(s)$



$dy(t)/dt = x(t)$

Laplace transform:

$s\,Y(s) = X(s)$

$H(s) = Y(s)/X(s) = 1/s$

Let $x(t) = 1$

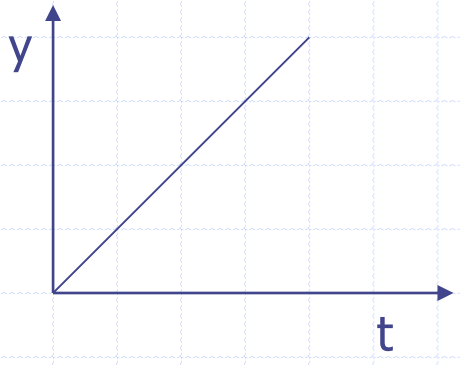
=>

$X(s) = 1/s$

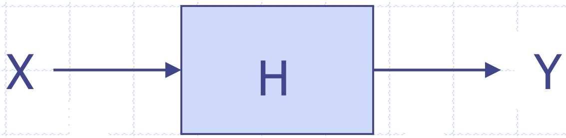
$Y(s) = H(s)\,X(s) = 1/s^2$

<=

$y(t) = t$



Control System Analysis



$$Y(s) = H(s)\,X(s)$$

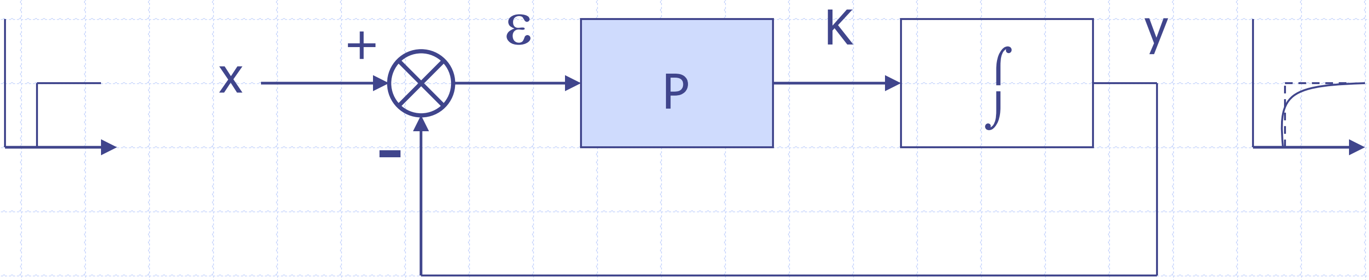
zeros

$$H(s) = \frac{Y(s)}{X(s)}$$

poles

Stability: Re(roots H(s)) < 0
Im(roots H(s)) small

Example: Rate Control (2)



$Y(s) = P\ H(s)\ (X(s) - Y(s))$
$Y(s) = (P\ H(s) / (1 + P\ H(s)))\ X(s) = H_{PC}(s)\ X(s)$

$H(s) = 1/s$
$H_{PC}(s) = (P/s) / (1 + P/s) = P / (s + P)$

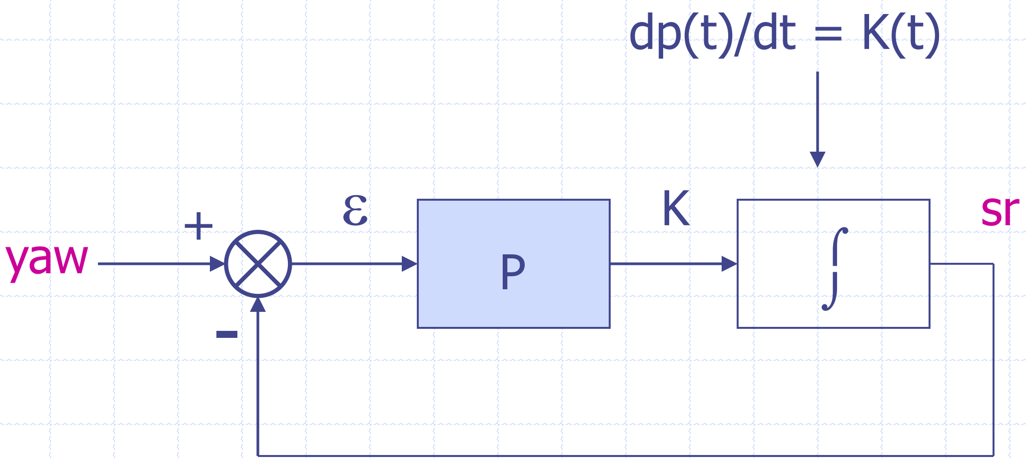
First-order system with time constant 1/P
(root: s = -P => Re < 0, Im = 0) so stable

Part IV: QR Control

- Instability Problem
- Cascaded P Control

Rate control using P controller

P controller for roll rate:



P < 1: useless control performance

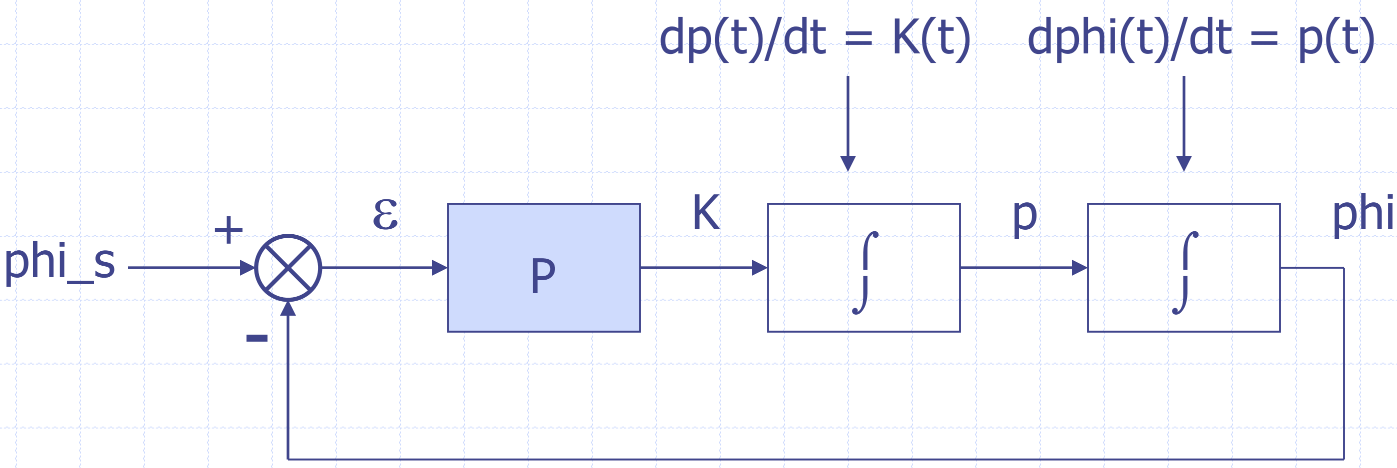
P ≥ 1: stable (for not too high P!)

Angle control using P contro

Laplace cheat sheet

- $L[a] = a/s$
- $L[a\,t] = a / s^2$
- $L[f+g] = F(s) + G(s)$
- $L[f'] = s\,F(s) - f(0)$
- $L[g(f)] = F(s)\,G(s)$

P controller for roll angle:



P < 1: useless control performance

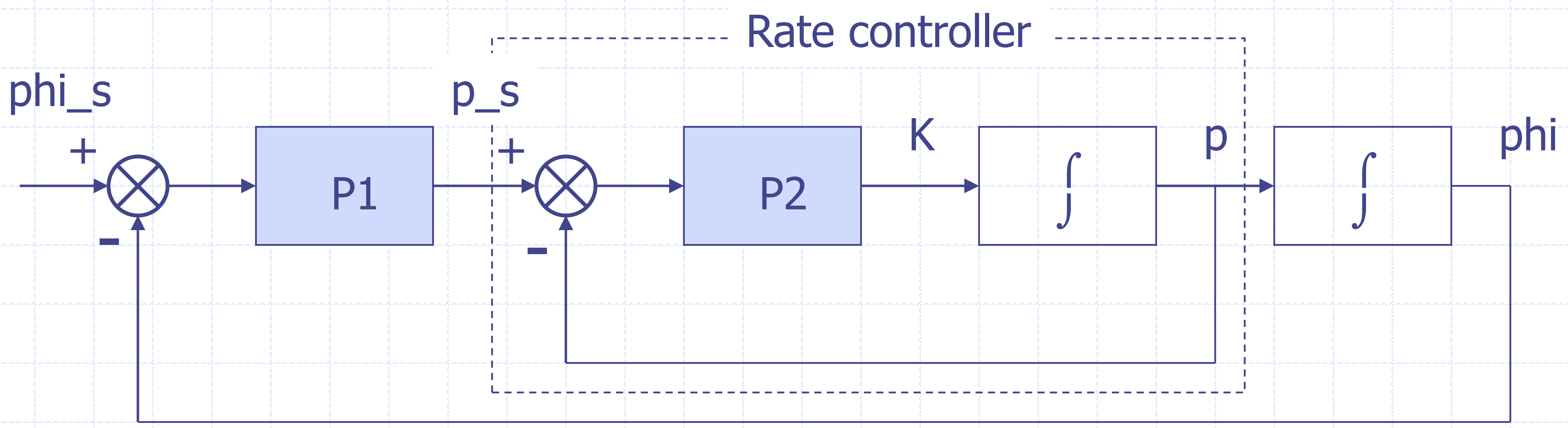
P ≥ 1: instability

Cascaded P control

Laplace cheat sheet

- L[a] = a/s
- L[a t] = a / s^2
- L[f+g] = F(s) + G(s)
- L[f ′] = s F(s) – f(0)
- L[g(f)] = F(s) G(s)

Embedded rate controller "neutralizes" one integrator



Cascaded P Controller: stable (for not too high P1 and P2!
and P2 ≥ 4 · P1)

Summary

- Feedback control offers many advantages
- Is ubiquitous (cars, planes, missiles, QRs ..)
- Potential stability problems
- Need control theory
- This was merely introduction into the field
- Get a feel by applying to QR!