Mechanism Design (for Multiagent Planning)

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Ch.9 on Introduction to Mechanism Design

Algorithmic Game Theory

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Social multiagent planning (1)

**Def.** Given a set of $n$ agents, a simple social multiagent planning problem is defined by a set \( \{(O_i,<_i) \mid 1 \leq i \leq n\} \), where
- \( s_0 \in S \) is a description of the initial state,
- \( O_i \) is the set of operations of agent \( i \),
- a **multiagent plan** \( a \in A \) is a DAG of operations of all agents, and
- \( <_i \in L \) is a linear transitive order over \( A \) (preference of agent \( i \)).

**Goal.** Find a multiagent plan that is “good” for every agent (social choice).

**Example.**

\[ O_1 = \{ \text{color_red} \} \quad \begin{array}{ccc} & >_1 & >_1 \\ & \text{block} & \text{red} \end{array} \in L \]

\[ O_2 = \{ \text{create_block} \} \quad \begin{array}{ccc} & >_2 & >_2 \\ \text{block} & \text{red} & \text{white} \end{array} \in L \]

**Q.** What is “good” for every agent in this example?
Social welfare functions (1)

Q. What is “good” for every agent in this example?
A. Use majority voting:

| 2 | 0 |
| 1 | 1 |

> 2 > 1 ∈ L

> 1 > 2 ∈ L

Def. Social choice function: given transitive orders, select winner: $L^n \rightarrow A$
For example:  

Def. Social welfare function: given transitive orders, social order: $L^n \rightarrow L$
For example:  

[TU Delft]
Mechanism design

**input:**
- private information agent 1
- private information agent 2
- ... private information agent n

**output:**
1. social welfare (preferences)
2. social choice (winner)

**private information:**
- preferences
- (assume possible actions known to everyone)

**mechanism:**
- social welfare function $F: \mathbb{L}^n \rightarrow \mathbb{L}$
- social choice function $f: \mathbb{L}^n \rightarrow \mathbb{A}$
Social welfare functions (2)

Q. What is “good” for every agent in this example?
A. Use majority voting:

Marquis de Condorcet (1785): majority vote does not work if $|A| > 2$. 

TU Delft
Q. What if every agent has the same order $<$ (with $a$ most preferred)?
A. Social choice should be this order $<$. Choose $a$ (unanimity).

Q. What if some irrelevant option $a$ is removed from $A$? For example:
$e > d > c > b > a$ 
$\text{d > c > b > e > a}$ 
$\text{b > d > c > e > a}$
A. Choice for $A \setminus \{a\}$ should be the same as for $A$ (monotonicity)
Def. Independence of irrelevant alternatives (IIA): relative social order of $a$ and $b$ is determined only by relative order of $a$ and $b$ in input

Q. A possible solution is “one agent determines every choice”. Like it?
A. No, no dictatorship, please.

Arrow’s theorem (‘51/’63): every transitive social welfare function $F: L^n \rightarrow L$ over $|A| > 2$ that satisfies unanimity and independence of irrelevant alternatives is a dictatorship.
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Pf. Given

- number of agents $n$,
- set of alternatives $A$, and
- social welfare function $F$ that satisfies unanimity and IIA.
- pairwise neutrality (each pair of alternatives is treated similarly)

Idea:
1. find “pivotal” dictator $i^*$ for $F$, $n$, $A$ (construction where $i^*$th player flips the result)
2. for any alternatives $c \neq d$, given all $>_i$, show that if $c >_{i^*} d$ then $c >_F d$

so $i^*$ is a dictator for $F$ and $F$ is a dictatorship.
Arrow’s theorem (‘51/’63): every transitive social welfare function $F : L^n \to L$ over $|A| > 2$ that satisfies unanimity and independence of irrelevant alternatives (IIA) is a dictatorship.

**Pf.** 1. Find “pivotal” dictator $i^*$ for $F$, $n$, $A$: For any $a \neq b \in A$, for any other relative order of other preferences.

- Construct preference profiles $\pi^i \in L^n$ where first $j \leq i$ players: $a > j b$

```
\begin{array}{ll}
\pi^0 & \pi^{i^*} & \pi^n \\
\ldots > 1 b > 1 \ldots > 1 a > 1 \ldots & \ldots > 1 a > 1 \ldots > 1 b > 1 \ldots & \ldots > 1 a > 1 \ldots > 1 b > 1 \ldots \\
\ldots > 2 b > 2 \ldots > 2 a > 2 \ldots & \ldots > 2 a > 2 \ldots > 2 b > 2 \ldots & \ldots > 2 a > 2 \ldots > 2 b > 2 \ldots \\
\ldots > n b > n \ldots > n a > n \ldots & \ldots > n a > n \ldots > n b > n \ldots & \ldots > n a > n \ldots > n b > n \ldots \\
\end{array}
```

\[\forall F, \exists \text{situation } \exists i^* \text{ that determines result of } b > a.\]
Arrow’s theorem (‘51/’63): every transitive social welfare function $F:L^n \rightarrow L$ over $|A|>2$ that satisfies 
unanimity and independence of irrelevant alternatives (IIA) is a dictatorship.

Pf. 2. For any alternatives $c \neq d$, given $>_i$, show that if $c >_{i^*} d$ then $c >_F d$:

- introduce “irrelevant alternative” $e$ (to create situation in profile $\pi^{i^*}$)
  - for $i < i^*$: $e >'_i \ldots$ on the top
  - for $i > i^*$: $\ldots >'_i e$ on the bottom
  - for $i^*$: $c >'_i e >'_i d$ (so $c >'_i d$)
- using pairwise neutrality (each pair of alternatives is treated similarly):
  - $c >'_F e$ follows from construction, replacing $a$ by $c$, and $b$ by $e$
  - $e >'_F d$ follows from construction, replacing $a$ by $e$, and $b$ by $d$
  - therefore $c >'_F d$ (in this specific situation)
- because $e$ should be irrelevant for $c$ and $d$, also $c >_F d$ (IIA).
Arrow’s theorem (3a)

Pf. 2. For any alternatives c ≠ d, given >_i, show that if c > i* d then c >_F d:
- introduce “irrelevant alternative” e (to create situation in profile π i*)
  - for i < i*: e >’i ... on the top
  - for i > i*: ... >’i e on the bottom
  - for i*: c >’i* e >’i* d (so c >’i* d)
Work around Arrow’s theorem

Arrow’s theorem (‘51/’63): every transitive social welfare function $F : L^n \rightarrow L$ over $|A| > 2$ that satisfies unanimity and independence of irrelevant alternatives is a dictatorship.

Q. How to work around this theorem?
A.
- only make choices between two alternatives (majority rule)
- relax “independence of irrelevant alternatives” requirement
  - used in most current voting systems (if candidate drops out...)
- restrict preference profiles
  - single-peaked preferences (Ch.10)
- relax social preference ordering (Amartya Sen)
  - no transitivity (only acyclicity)
- focus on social choice functions?
Social choice functions (1)

Def. A social choice function $f$ is **monotonic** iff outcome (say $a$) does not change if relative order of alternatives ($\neq a$) is changed in input. (similar to IIA)

**Muller-Satterthwaite (’77):** *every* social choice function $f: L^n \rightarrow A$ over $|A| > 2$ that satisfies *unanimity* and **monotonicity** is a **dictatorship**.

**Pf.** Similar to Arrow’s.
**Social choice functions (2)**

**Def.** A social choice function \( f \) can be **strategically manipulated** if an agent \( i \) with \( a <_i a' \) can ensure that \( a' \) gets chosen instead of \( a \) by lying.

**Def.** If \( f \) cannot be strategically manipulated it is **incentive compatible** (truthful).

**Q.** Want mechanisms that can be strategically manipulated? Why (not)?

**A.** No.

- Social choice will be sub-optimal
- Manipulation is difficult for agents. Waste of time and resources.

**Gibbard-Satterthwaite (’73):** *every* transitive social choice function \( f:L^n \rightarrow A \) over \(|A|>2\) that satisfies **incentive compatibility** is a dictatorship.

**Pf. (Sketch, by contradiction)** Suppose such an \( f \) is not a dictatorship.
Construct social welfare function \( F \) by using choice by \( f \) repeatedly. \( F \) is then not a dictatorship. Contradiction with Arrow’s theorem.
Def. Given a set of $n$ agents, a social multiagent planning problem is defined by a set \( \{(O_i, <_i) \mid 1 \leq i \leq n\} \), where
- $s_0 \in S$ is a description of the initial state,
- $O_i$ is the set of operations of agent $i$,
- a multiagent plan $a \in A$ is a DAG of operations of all agents, and
- $<_i \in L$ is a linear transitive order over $A$ (preference of agent $i$).

Goal. Find a multiagent plan that is “good” for every agent (social choice).

Q. Is it possible to find a mechanism to solve this problem for self-interested agents (where $<_i$ is the private information)? How/why not?

A. No. Gibbard-Satterthwaite (’73) say:
- Agents will have incentive to lie about preferences (or operations), or
- social choice will be a dictatorship.

Maybe introduce money or tax to incentivize agents...
Mechanisms with money

**Assumption.** Every agent \(i\) has value \(v_i(a)\) for each alternative \(a\): \(v_i : A \rightarrow \mathbb{N}\)
so \(v_i \in V\) instead of preference order \(<_i \in L\)

**Simple multiagent planning setting:** agent \(i\) has one unique favorite plan \(a_i\)
for all \(i\), for all \(a\): \(v_i(a) = 0\) if \(a \neq a_i\)
choose plan \(f(v_1, \ldots, v_n) = \arg\max_i v_i(a_i)\)

Knowing this, agents bid as high as possible (strategic manipulation).

**Q.** How to make this protocol incentive compatible?

**A.** Use a Vickrey auction.

**Def.** Vickrey’s second-price auction:

- winner = player \(i\) with highest declared value \(v_i\), and
- price = second-highest declared bid.

**Vickrey’s theorem (’61):** this auction cannot be strategically manipulated.
William Vickrey (1914-1996)

- Nobel prize in economic sciences at October 8th 1996
- heart attack three days later

- studied road transportation in Washington, D.C. in 1959
  - no tollbooths, but small radio transmitters
  - built rudimentary computer in his home with radio receiver
  - small radio transmitter (for under $3) under the hood of his car
  - printout times his own car went up or down his driveway

- printouts showed Vickrey rarely used his car:
  - took train into Manhattan
  - “commuted” the blocks from the station and across Columbia’s campus to his office on roller skates
VCG mechanisms (1)

Introduce payments $p_i$

**Def. mechanism** = social choice $f: V^n \rightarrow A$ and $\forall i$ payment $p_i: V^n \rightarrow N$

**Def.** A mechanism $(f, p)$ is incentive compatible iff no player can win by lying if all others tell the truth. Formally: $\forall i \forall (v_i, v_{-i}) \in V^n \forall v_i' \in V_i$

- let $a = f(v_i, v_{-i})$ and $a' = f(v_i', v_{-i})$
- $v_i(a) - p_i(v_i, v_{-i}) \geq v_i(a') - p_i(v_i', v_{-i})$

**Q.** How to realize incentive compatibility?

**Def. Vickrey-Clarke(’71)-Groves(’73) VCG mechanism:** assign payments $p_1, \ldots, p_n$ to each agent such that

- $f$ maximizes social welfare: $a = f(v_1, \ldots, v_n) \in \arg \max_{a' \in A} \sum_i v_i(a')$
- payment $p_i$ does not depend on $v_i$ but on values for all other players:
  $p_i(v_1, \ldots, v_n) = -\sum_{j \neq i} v_j(a) + h_i(v_{-i})$

**VCG theorem:** every VCG mechanism is incentive compatible.
VCG theorem

**VCG theorem**: every VCG mechanism is incentive compatible.

**Pf.** Idea: follows from definitions of VCG and incentive compatible.

Let $i, v_{-i}, v_i$ and $v_i'$ be given. Let $a = f(v_i, v_{-i})$ and $a' = f(v_i', v_{-i})$.

We show that $v_i(a) - p_i(v_i, v_{-i}) \geq v_i(a') - p_i(v_i', v_{-i})$.  (incentive compatible)

$f$ maximizes social welfare:  

\[
f(v_1,\ldots,v_n) \in \arg \max_{a \in A} \sum_i v_i(a)
\]

- so $\sum_j v_j(a) \geq \sum_j v_j(a')$
- so $v_i(a) + \sum_{j \neq i} v_j(a) \geq v_i(a') + \sum_{j \neq i} v_j(a')$
- so $v_i(a) + \sum_{j \neq i} v_j(a) - h_i(v_{-i}) \geq v_i(a') + \sum_{j \neq i} v_j(a') - h_i(v_{-i})$

and since (VCG mechanism)

\[
p_i(v_i, v_{-i}) = p_i(v_1,\ldots,v_n) = -\sum_{j \neq i} v_j(a) + h_i(v_{-i}) = p_i(v_i', v_{-i})
\]

it holds that

\[
v_i(a) - p_i(v_i, v_{-i}) \geq v_i(a') - p_i(v_i', v_{-i})
\]
VCG mechanisms (2)

Q. When do agents want to take part in such a (VCG) mechanism?
A. If they get positive utility, i.e. if
\[ v_i(a) - p_i(v_i, v_{-i}) = v_i(a) + \sum_{j \neq i} v_j(a) - h_i(v_{-i}) \geq 0 \]
Def. This is called individual rationality.

Def. Take \( h_i(v_{-j}) = \max_{b \in A} \sum_{j \neq i} v_i(b) \), the Clarke pivot payment.

So the utility of agent i is
\[ v_i(a) + \sum_{j \neq i} v_j(a) - \max_{b \in A} \sum_{j \neq i} v_i(b) \]
This is its marginal contribution!

Fixing \( b = \text{arg max}_{b \in A} \sum_{j \neq i} v_i(b) \) the utility of i is:
\[ = v_i(a) + \sum_{j \neq i} v_j(a) - \sum_{j \neq i} v_i(b) \]
\[ = \sum_j v_j(a) - \sum_{j \neq i} v_i(b) \]
\[ \geq \sum_j v_j(a) - \sum_j v_i(b) \quad \text{(because } v_i(b) \geq 0) \]
\[ \geq 0 \quad \text{(because } a \text{ maximizes } \sum_j v_j(a)) \]
Lem. VCG mechanism with the Clarke pivot payment is individually rational.
VCG for multiagent planning with money

**Def.** Given a set of \( n \) agents, a **multiagent planning problem with money** is defined by a set \( \{(O_i, <_i) | 1 \leq i \leq n\} \), where
- \( s_0 \in S \) is a description of the initial state,
- \( O_i \) is the set of operations of agent \( i \),
- a multiagent plan \( a \in A \) is a DAG of operations of all agents, and
- function \( v_i : A \rightarrow \mathbb{N} \) describes private value for \( i \) of each alternative

**Goal.** Find a multiagent plan \( a \in A \) that optimizes \( \Sigma_i v_i(a') \).

**Q.** Is it possible to find a mechanism to solve this problem for self-interested agents (if \( v_i \) is private information) ? How/why not?

**A.** Yes, using a VCG-based mechanism, but:
1. we retrieve some “tax”; what to do with that?
2. social choice must be optimal, general planning is PSPACE-complete
**Shortcomings of VCG**

**Def.** A mechanism is **budget-balanced** iff $\Sigma_i p_i(a) = 0$.

**Lem.** VCG is not budget-balanced.

**Myerson-Satterthwaite (’83):** Efficient budget-balanced mechanisms are impossible in general (even in bilateral trade).

But there are some **redistribution mechanisms**, giving back part of payment.

**Lem.** VCG requires **exact** optimization (approximation loses IC)

**Nisan-Ronen (’07):**

- **possible** for some problems where approximation solution is maximal in range, i.e. max & not depending on agent’s types (eg multi-unit auctions)
- **not possible** for others, eg, for any cost-minimization allocation problem any sub-optimal VCG-based mechanism is degenerate (i.e. can have solutions arbitrarily far from optimal).
Summary / future directions

Multiagent planning for self-interested agents

- as a social choice: difficult, look for relaxations / special cases
  - e.g. when preferences are “single-peaked”
- as a VCG mechanism:
  - what to do with the money?
  - bad results if not optimal (“cost minimization problem”)
- other mechanisms with money?
  - indications are, sometimes VCG is only one (Robert’s)
Your applications...

Q. Example of problem including multiple participants?
   ▪ Adriaan: agents coordinate use of edges in s-t path

Q. How can (a simpler version of) this problem be seen as a one-shot truthful mechanism design problem?
   ▪ If it is natural to use payments, can you apply the VCG mechanism?
   ▪ If you are just considering preferences, what are the consequences of Arrow's and Gibbard-Satterthwaite's theorem for your problem?
Prop. Direct revelation principle (§ 9.4.3).
if \( \exists \) arbitrary mechanism implementing \( f \) in dominant strategies
then \( \exists \) incentive compatible direct revelation mechanism implementing \( f \)
with the same payments.
Pf. (Sketch) simulate dominant agent strategies inside mechanism.

So we can focus on direct revelation mechanisms.
Mechanisms with money

Robert’s theorem (’79): When domain of preferences is unrestrictive, the only incentive compatible mechanism is a VCG mechanism. In this case the social choice function is an affine maximizer.

Def: affine maximizer: for some subrange $A' \subseteq A$, agent weights $w_i$, the social choice function is

$$f(v_1, \ldots, v_n) \in \arg\max_{a \in A'} (c_a + \sum_i w_i v_i(a)).$$