A Linear Programming based Satisfiability solver using a new Horn-driven search tree design

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We will present an algorithm for the satisfiability problem, which finds its origin in the Integer Programming area, and therefore will also generalize to more general constraint programming problems. This algorithm is based on single-lookahead unit resolution (Franco(1997)), linear programming and a new search tree design based on a tree design by Van Maaren and Dang (2002). The special aspect of the tree is that it is not a binary search tree. The advantage of our algorithm over a standard integer programming approach, is that we need to solve a linear program only in a very limited number of nodes in the search tree. In every node in the search tree we first apply single-lookahead unit resolution. The unit propagation algorithm we used in our implementation is based on the watched literal strategy. Only in case the unit resolution does not lead to the conclusion that the formula in the node is unsatisfiable or has a satisfying assignment, we solve a linear program. The solution of the linear program is used to split the part of the search into subparts. This splitting aims for getting a formula close to a Horn formula.

Horn clauses are clauses in which at most one variable occurs in negated form. Therefore, formulae with only Horn clauses of size two or larger are always satisfied if all variables are set to true. After unit resolution, all clauses have length two or more. The linear programming algorithm finds a feasible solution that is as close as possible to the all-ones vector. An almost Horn formula will therefore yield linear programming solutions that are closer to an integer-valued solution. By construction, conflicts are likely to be close to the linear programming solution. Preprocessing is applied to make formulae closer to Horn formulae. Below we give a brief overview of our algorithm

1. Apply single look-ahead unit propagation in order to simplify the formula
2. Apply an heuristic to make the formula more Horn-like
3. If the formula is not proved to be unsatisfiable or satisfiable, transform the simplified CNF-formula to a set of linear constraints
4. Solve the linear program, this gives a solution vector ω
5. Use ω to partition the search space.

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