Question 1

(3 points)

Coverage testing is closely related to determining the WCET of a program. One of the most prominent criteria adopted by industry is the Modified Condition / Decision Coverage (MC/DC) criterion. It enforces thorough testing of if-then-else (branching) statements.

```c
if ((flag_a || flag_b) && flag_c) {
```

(a) describe what properties should be checked on conditional expressions like the one above to obtain MC/DC certification.

(b) give a minimum set of test patterns (variable/value pairs) that is needed to verify MC/DC.

(a) every sub-condition (e.g. flag_a, flag_b) must be shown to affect the decision independently (of all other sub-conditions).

(b) flag_a flag_b flag_c

<table>
<thead>
<tr>
<th></th>
<th>flag_a</th>
<th>flag_b</th>
<th>flag_c</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>
Question 2  

(6 points)

2.1) Consider a set of five tasks simultaneously activated at time $t = 0$. The computation times and deadlines of these tasks are given below:

<table>
<thead>
<tr>
<th>$\tau_1$</th>
<th>$\tau_2$</th>
<th>$\tau_3$</th>
<th>$\tau_4$</th>
<th>$\tau_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_i$</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>$d_i$</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

(a) (1.5 points) Is there a feasible EDF schedule for this set of tasks? What is the maximum lateness?

(b) (1.5 points) If we add the precedence constraints described below, is there a feasible schedule?

\[ \tau_1 \rightarrow \tau_2 \]
\[ \tau_1 \rightarrow \tau_3 \]
\[ \tau_2 \rightarrow \tau_4 \]
\[ \tau_2 \rightarrow \tau_5 \]

Solution part (a):

* Sort $d_i$ in increasing order.

max lateness = 0

2.11) Determine if the sets of periodic tasks below are schedulable under the Rate Monotonic algorithm

(a) (1 point)

<table>
<thead>
<tr>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_i$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$T_i$</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

(b) (1 point)

<table>
<thead>
<tr>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_i$</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$T_i$</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

(c) (1 point)

<table>
<thead>
<tr>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_i$</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$T_i$</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

Using the Hyperbolic bound:

\[
\left( \frac{2}{6} + 1 \right) \left( \frac{2}{8} + 1 \right) \left( \frac{2}{11} + 1 \right) = 1.96 \leq 2 \]

scheduled

\[
\left( \frac{3}{6} + 1 \right) \left( \frac{2}{8} + 1 \right) \left( \frac{1}{20} + 1 \right) = 1.9688 \leq 2 \]

scheduled

\[
\left( \frac{2}{6} + 1 \right) \left( \frac{3}{9} + 1 \right) \left( \frac{4}{10} + 1 \right) = 2.48 \geq 2 \]

we can't say much.

\[
\frac{2}{C_i} = 1.06 \]

NOT SCHEDULED
Question 3

(26 points)

Given the following preemptable periodic tasks scheduled based on the Rate Monotonic Algorithm:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_1 )</td>
<td>2 7</td>
</tr>
<tr>
<td>( \tau_2 )</td>
<td>3 11</td>
</tr>
<tr>
<td>( \tau_3 )</td>
<td>7 19</td>
</tr>
</tbody>
</table>

(a) (8 points) Is there a feasible schedule for these tasks?

(b) (8 points) Derive the Best Case Response time for all tasks.

(c) (10 points) Can a fourth task be added to this set while still guaranteeing schedulability? If your answer is “No”, provide a proof. If you answer is “Yes”, provide a computation time \( C_i \) and period \( T_i \) for the fourth extra task.

To check schedulability, first we need to check the critical instance.

(a) We need to check best case conditions: WCRF

(b) We need to check best case conditions: BCRF

(c) Several potential answers: (See diagram on next page)

One of them: idle time in worst case analysis occurs between 53 and 55.

Then we can have \( C_1 = 3 \) and \( T_1 = 55 \) of \( C_1 = 1 \) and \( T_1 = 55 \)

or \( C_1 = 1 \) and \( T_1 = \text{lcm}(7, 11) = 77 \times 11 \times 19 \)
Another option for part c):

→ Since we know that the task set is schedulable for the three tasks, then we know that we have some slack because $\sum U_i < 1$.

→ Then we could add a fourth task with $C_4 = 1$ and $T_4 = \text{hyperperiod}$.

Important note: if you would like to add a fifth task then first you will need to check that the 4 tasks are schedulable, and then, apply the same approach as above. Depending on the interplay of the $T_i$'s and $C_i$'s, you may reach $\sum U_i = 1$ or not.
Question 4

Given the following preemptable periodic tasks released at time $t=0$:

<table>
<thead>
<tr>
<th>$C_i$</th>
<th>$D_i$</th>
<th>$T_i$</th>
<th>$T_i - D_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>1</td>
<td>2.5</td>
<td>3</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>2</td>
<td>5.5</td>
<td>7</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>3</td>
<td>8.5</td>
<td>10</td>
</tr>
</tbody>
</table>

(a) (12 points) Under EDF, the schedulability of these tasks needs to be verified in the time interval $[0, L]$. First derive $L$ and then verify the schedulability of the tasks in the period $[0, L]$.

(b) (8 points) Is there a feasible schedule for these tasks under the Deadline Monotonic algorithm?

(c) (8 points) In general, given a set of tasks that is schedulable under EDF, what conditions—in terms of the $C_i$'s, $D_i$'s and $T_i$'s—could make the same set of tasks unschedulable under the Deadline Monotonic Algorithm? Describe at least two conditions.

(d) (4 points) For the tasks shown above, if the $D_i$'s take the same values as the $T_i$'s, what EDF method can be used to determine if the tasks are schedulable?

\[ H = 3 \times 7 \times 10 = 210 \]
\[ D_{\max} = 8.5 \]
\[ L^* = \frac{\sum_{i=1}^{n} (T_i - D_i) U_i}{1 - U} = 12.91 \]

Yes it is schedulable because for all points $g(0, L) \leq L$.

Solution (a)

\[ \min (H, \max (0_{\max}, L^*)) \]

\[ H = 3 \times 7 \times 10 = 210 \]

\[ D_{\max} = 8.5 \]

\[ L^* = \frac{\sum_{i=1}^{n} (T_i - D_i) U_i}{1 - U} = 12.91 \]

Yes it is schedulable because for all points $g(0, L) \leq L$.

\[ g(0, 2.5) = 4 ; g(0, 5.5) = 4 ; g(0, 8.5) = 8 \]

\[ g(0, 11.5) = 9 ; g(0, 12.5) = 11 ; \]

\[ g(0, L) = 12.91 \]

(b) $\tau_1$ $\tau_2$ $\tau_3$ $\tau_1$ $\tau_2$ $\tau_3$

\[ \text{missed deadline} \]

\[ \text{we need to consider worst case conditions, i.e. critical instance.} \]
(C) Under EOF assume that task are schedulable.

\[ Ti \rightarrow D_i \]

- Ti has a higher priority than Tj.

\[ D_j \uparrow \]

What conditions would move block 1 before block 2?

- Be more prone to:
  * Longer relative deadlines of higher priority tasks (Di).
  * Longer computation times of higher priority tasks (Ci).
  * High number of higher priority tasks.

(a) Simple utilization:

\[ \sum_{i=1}^{n} U_i \leq 1 \]

\[ \frac{1}{3} + \frac{2}{4} + \frac{3}{10} \leq 1. \]
Question 5  

Given the following task set

<table>
<thead>
<tr>
<th></th>
<th>$C_i$</th>
<th>$D_i$</th>
<th>$T_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>2</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>1</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>3</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

(a) (8 points) Assuming that the tasks are preemptable, is the task set schedulable under the Deadline Monotonic Algorithm?

(b) (12 points) Assuming that the tasks are non-preemptable, is the task set schedulable under the Deadline Monotonic Algorithm?

(c) (5 points) If the deadline of task $\tau_3$ is changed from 6 to 5.5, describe the steps you will need to take to verify the schedulability of the task set. Do not solve the problem, simply state the steps and the reasoning behind them.

\[ (a) (1) \quad R_2 = C_2 = 1 \leq D_2 \checkmark \]
\[ (2) \quad R_1^0 = \sum_{i=1}^{2} C_i = 3 \leftarrow \]
\[ R_1' = C_1 + \left\lfloor \frac{R_2^0}{T_2} \right\rfloor C_2 = 2 + \left\lfloor \frac{3}{8} \right\rfloor 1 = 3 \leq 5 \checkmark \]
\[ (3) \quad R_3^0 = \sum_{i=1}^{3} C_i = 6 \leftarrow \]
\[ R_3' = C_3 + \left\lfloor \frac{R_2^0}{T_1} \right\rfloor C_1 + \left\lfloor \frac{R_2^0}{T_2} \right\rfloor C_2 \]
\[ = 3 + \left\lfloor \frac{6}{6} \right\rfloor 2 + \left\lfloor \frac{6}{8} \right\rfloor 1 = 6 \leq 6 \checkmark \]

The conditions for the three tasks are satisfied, hence it is schedulable.
(b)

Blocking

T3

T2

T1

OK

missed deadline.

(c) Step 1
First obtain number of instances \( N_i = \left[ \frac{L_i}{T_i} \right] \)

Step 2
Then use \( SOK = B_i + (k-1)C_i + \sum_{h=1}^{i-1} \left( \left[ \frac{S_i}{T_h} \right] + 1 \right)C_h \)

For part b) we should state given that \( D_i < T_i \) is AND given that the task set is schedulable under preemption we only need to check the first instance \( (k=1) \).
- First we need to obtain the blocking time \( B_i \).

\[
B_i = \max \{ C_j \} \quad \text{for } p_j < B_i
\]

For Task 1: \( B_1 = \max \{ 3 \} = 3 \)
For Task 2: \( B_2 = \max \{ 2, 3 \} = 3 \)

Then we do worst case blocking analysis as shown in the above diagram.

For part c) we need to state that given that the task set is not schedulable under preemption we need to check \( k > 1 \) instances.
Then we can follow the two steps shown above.
Question 6 (8 points)

Considering the set of elastic tasks below, to be scheduled by EDF:

<table>
<thead>
<tr>
<th></th>
<th>$C_i$</th>
<th>$T_i^{min}$</th>
<th>$T_i^{max}$</th>
<th>$E_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>7</td>
<td>13</td>
<td>19</td>
<td>2</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>11</td>
<td>17</td>
<td>27</td>
<td>4</td>
</tr>
</tbody>
</table>

(a) (8 points) Considering that these tasks are not schedulable with the minimum period, derive the periods $T_i$'s that lead to a total utilization of 1 using the Elastic Period method.

\[ U_i = U_{i-1} - (U_0 - U_d) \frac{E_i}{E_T} \]

\[ U_1 = \frac{7}{13} - \left( \frac{7}{13} + \frac{4}{17} \right) - 1 \frac{2}{6} = 0,47 \]

\[ U_2 = \frac{11}{17} - \left( \frac{7}{13} + \frac{4}{17} \right) - 1 \frac{4}{6} = 0,52 \]

\[ T_1 = \frac{7}{0,476} = 14,7 \]

\[ T_2 = \frac{11}{0,52} = 21,2 \]