Fault detection in non linear systems with parameter uncertainty: a Particle Filter approach.

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ABSTRACT
Model-based diagnosis provides a systematic methodology to developing fault diagnosis of complex systems, but the key to robust detection and correct isolation of faults is an accurate and precise model of system dynamic behavior. More recently, structural methods have been extended to developing accurate models of nonlinear systems, but given the numerical issues associated with generating and tracking non linear behaviors accurate parameter estimation is still a difficult task. The difficulty of the overall task is further compounded when the sensor measurements are noisy. We have been working on Dynamic Bayesian Networks (DBNs) and Particle Filtering methods for dealing with noise in measurements and uncertainty in the system model. In this paper, we extend the notion of parameter uncertainty, using the methods developed in the bond graph modeling framework. We extend a DBNs Particle Filtering method to deal with parameter uncertainties during the tracking and fault detection tasks. Robust fault detection of abrupt and incipient faults has been performed. We have run experiments on a nonlinear high-order spring mass system to demonstrate the effectiveness of this approach.

1 INTRODUCTION
Increasing complexity of current dynamic systems, together with increasing demands on their reliability from the very beginning of their life cycle, make model based diagnosis the preferred methodology to develop robust diagnosis systems, but this depends very much on the availability of accurate and precise models of dynamic system behavior. Model based diagnosis exploits analytical redundancy of the system generating residuals by comparing the outputs of the system with the predictions of the model. In a fault free system, residual values are theoretically zero. Under ideal conditions, non-zero residual values are indicative of a system failure. Consequently, robust residual generation is an important element on model based diagnosis.

A number of approaches, such as parity space, state observer design, and analytical redundancy relations have proved to be effective, and, furthermore, have been shown to be theoretically equivalent (Gertler, 1998).

Residual generation becomes a harder problem for nonlinear systems because accurate and precise models are harder to build, closed form solutions are harder to derive, and numerical solutions often plagued by convergence problems. Optimal state estimation is not possible for general nonlinear systems. Analytical redundancy may be reduced due to the presence of non-invertible relations, which also may prevent variable elimination and the application of parity space approach. Strong nonlinearities may make impractical computer simulation of the system equations, which usually have no analytical solution. Observability of the system, necessary to estimate its states and parameters, may be lost for certain singular input values, making residual generation extremely difficult for inputs close to those singular values. For general nonlinear systems, residual sensitivity becomes an issue that is hard to address using analytical and numerical methods. To cope with this problem, structural approach to system analyses have been proposed as alternative approaches. We have adopted this idea, and used the topological Bond Graph (Karnopp et al., 2000; Samantaray and Bouamama, 2008) approach to model nonlinear systems.

Fault diagnosis of nonlinear systems becomes even harder when the system model is uncertain. In this paper, we assume that the structure of the model is known and we focus on the problem when model parameters are uncertain. This is an important, realistic scenario because many real systems are nonlinear, and given the numerical convergence issues associated with non linear models, their parameter values may only be known approximately. From the point of view of tracking system behavior and residual generation for nonlinear systems, small changes in system parameter values may produce widely varying dynamic behavior. Besides, in reality, system measurements are noisy. Therefore, the key issue is how we can employ uncertain reasoning methods to track dynamics of be-
behavior close enough to generate accurate residuals for fault detection and isolation under uncertainty in the system parameters.

At present, there are two main techniques to track the behavior of nonlinear systems on noisy environments: Unscented Kalman Filter (UKF) and Particle Filtering (PF). UKF (Julier et al., 2004) usually shows more precise behavior than the classic Extended Kalman Filter (Chatzi and Smyth, 2009). PF (Arulampalam et al., 2002) has the advantage of generality because it applies to non-Gaussian distributions. It is also usually more precise, although at a higher computational cost. In this paper we have opted for the Particle Filter approach. Our approach is, therefore, to combine particle filtering with modeling mechanisms that incorporate parameter uncertainties into the dynamic system model. We adopt a method developed by (Kam and Dauphin-Tanguy, 2005; Djeziri et al., 2007) that introduces parameter uncertainty as an additive phenomena in system Bond Graph (BG) models.

The rest of the paper is organized as follows. Section 2 discusses uncertainty measurement and model uncertainties, and specifically discusses the method for modeling parameter uncertainty in BG models. Section 3 discusses the particle filtering approach for tracking dynamic system behavior. Section 4 presents experimental results for tracking behavior of a nominal complex order non linear spring mass system. Section 5 shows some preliminary fault detection and residual generation results with abrupt and incipient faults using our approach. Section 6 presents the conclusion of this study.

2 MODELING PARAMETER UNCERTAINITIES

2.1 Theoretical approach

Typically uncertainties when using an observer to track dynamic system behavior can be attributed to two sources: (1) uncertainty in the model, and (2) uncertainty in the measurements (Gelb, 1979). Uncertainties in measurements, attributed to sensors, are typically modeled as zero mean, Gaussian noise, and the key is to estimate the unknown variance in the noisy measurements to increase the robustness for fault detection while tracking dynamic system behavior. This problem has been addressed for linear systems using Kalman filters, and nonlinear systems using methods such as the Extended Kalman filter, Unscented Kalman filter, and Particle filtering methods. In recent work, we have focused on the use of Particle Filtering methods for tracking noisy measurements in system behavior (Roychoudhury et al., 2008). Modeling uncertainties can be attributed to: (1) structural uncertainty in the system model, i.e., the system equation form or all of the causal relations between system variables may not be explicitly known, and (2) parameter uncertainty, where the system structure is known, but the values of the parameters of the model may not be known. In our work, we assume that the structure of the system is known, but the parameters of the model may be uncertain.

Typically, parameter uncertainty is handled by assuming the parameter value comes from a known interval, but the exact value of the parameter is unknown. We make this assumption in our work, and adopt a method developed by Kam and Dauphin-Tanguy and others (Kam and Dauphin-Tanguy, 2005; Djeziri et al., 2007) in the bond graph framework to model parameter uncertainties. The model created using this approach will be used to generate data for the experiments, what we called later ”simulated data”. We briefly discuss bond graph modeling of physical systems, and then discuss the parameter uncertainty approach.

2.2 Bond Graph Modeling and Running example

Bond Graphs (BGs) represent a multi-domain topological modeling approach for physical systems that captures energy exchange pathways between physical processes. The processes are represented as generic elements, such as energy storage (C and I), dissipation (R), transformation (GY and TF), source (Se and Sf), and sensor (De and Df) elements. The connecting edges, called bonds, represent energy pathways between the elements. Each bond, has effort and flow variables associated with it, such that effort × flow defines the power transferred through the bond. Sets of elements are connected to each other through idealized 0— and 1—junctions, which represent parallel and series connections, respectively. Nonlinearities in the BG model are represented by making the element parameter values functions of effort and flow values in the BG or as a function of external variables.

Figure 1 represents a spring-mass-damper model with a nonlinear resistance. Figure 2 represents the BG model of the system. There is an input force acting on the mass, represented as Se1. The spring, mass, and damper share a common velocity, and are, therefore, connected by a 1 junction. The velocity is measured by sensor Df : f2. The inertial element has a constant mass M1, the capacitance is given by spring constant K1 and the nonlinear damping coefficient, which is a function of the velocity of the mass is represented by the function f(x) = f2 × b1 + b0, where f2 is the common velocity of the system, b1 is a multiplicative constant, and b0 represents the initial value of the damping coefficient.

The system used to run the experiments presented...
in sections 4 and 5 is a more complex version of this system. It is discussed in Section 4.

2.3 Parameter Uncertainty in Bond Graph modeling

![Parameter Uncertainty Representation for R elements using the simplified approach. a) shows the structure when the resistance is in resistive causality, b) shows the structure when the resistance is in conductance causality.](image1)

![Parameter Uncertainty Representation for C and I elements in integral causality using the simplified approach. a) shows the structure for a capacitance element, b) shows the structure for an inductance element.](image2)

![Bond graph of the simple spring - mass - damper system with uncertainty in the capacitance K1.](image3)

As discussed earlier, model-based diagnosis approaches require accurate and precise system models, but nonlinearities and lack of sufficient data about system dynamics makes it hard to estimate the component parameters accurately. Kam and Dauphin-Tanguy and others (Kam and Dauphin-Tanguy, 2005; Djäziri et al., 2007) have developed methods to represent parameter uncertainties within the bond graph framework. In this first method, the uncertainty associated with a parameter $\theta$ is modeled as an absolute additive uncertainty, $\Delta \theta$, i.e., $\theta = \theta_0 + \Delta \theta$, and this uncertainty is modeled as a physical component in the bond graph framework. For example, an uncertain resistance parameter, $R = R_0 + \Delta R$ is represented by two elements $R_0$ and $\Delta R$ connected to a 1 junction. Depending on the situation, the causality of the $\Delta R$ bond can be the same as the causality of the $R_0$ bond as shown in Figure 3 (a) and (b)). We assume the $C$ and $I$ elements are always in integral causality. This implies that the corresponding $\Delta C$ and $\Delta I$ elements will always be in derivative causality on 0 and 1 junctions, respectively. The corresponding bond graph representation for the uncertain $C$ and $I$ elements are also shown in Figure 3 (c) and (d), respectively.

The second method uses an additive model for uncertainties. It has been proposed by Kam and Dauphin-Tanguy (Kam and Dauphin-Tanguy, 2005) and the uncertainty is modeled as an additive effect. To achieve this, the nominal (or expected) parameter value, $\hat{R}$, is modeled using power bonds, and the uncertainty value $\Delta R$ is modeled using signal links with a sensor element and a modulated source element through $\delta_R$, with $\delta_R = \frac{\Delta R}{R_0}$. Similarly for $C$ and $I$ parameters introducing $\delta C$ and $\delta I$. The structures used to model uncertainty with this approach in R elements are shown in Figure 4. Figure 5 shows the structure to add uncertainty in C and I elements. Uncertainty added to the spring element of the spring-mass-damper system is shown in Figure 6.
tion given the new state particles. The actual distribution followed by the system states is not always known, instead of this, another known distribution is used. It is easier to sample from the second (proposed) distribution. It is also better if the proposed distribution is close to the actual.

3.1 Using Particle Filters to cope with Parameter Uncertainty

A Particle Filter approach is used to simulate the system but some modifications have been added to deal with uncertainty in an efficient way.

A model is used to represent the system. The ideal or nominal model of the system describes it when parameters have their nominal values. We assume that we know the nominal value of the parameters and that the system structure does not change. We assume that the actual parameter values, although unknown, are close to their nominal values, and can be found in the interval $[P_n - \delta, P_n + \delta]$, being $P_n$ the nominal value of a parameter and $\delta$ the level of uncertainty in the parameter.

Assuming there is only uncertainty in the parameters and there is not uncertainty in the system structure we run several models with different values for the parameters using a particle filter approach and we finally work out the average of the estimation of all of them. For each model, the values for the parameters are sampled considering a uniform distribution over an interval. The interval, as it has been previously said, is built using a percentage over the nominal value for the parameter, $[P_n - \delta, P_n + \delta]$, where $\delta$ is the percentage of uncertainty.

Basically, an interval is built for each parameter with uncertainty. Considering a uniform distribution over the numbers of each interval, values for the parameters are sampled. N models are built using the previous method and each of those N models is run using a DBN with a particle filter approach. For each model, M particles are simulated and at the end, the estimation for the system behavior is the average of the estimation of each model. Algorithm 1 present a pseudocode for the process described.

**Algorithm 1** Particle filter approach sampling values for parameters with uncertainty.

1: Build intervals for all parameters with uncertainty. $[P_n - \delta, P_n + \delta]$. 
2: for $i = 1$ until $\text{numModels}$ do
3: Sample value for each parameter with uncertainty. Uniform distribution over the intervals.
4: Run a PF using those sampled values for the parameters.
5: end for
6: The estimation is the average of the $\text{numModels}$ estimations generated for the models.

4 CASE STUDY I: TRACKING NON FAULTY BEHAVIOR

4.1 Non Linear Spring-Mass-Damper System

The system used in the experimental study is a spring-mass-damper system with non linear resistances. It is presented in Figure 7. Figure 8 shows the bond graph model of the system. All the resistances in the system are modulated, the formula to calculate their values is $r_i = f_i \ast k_i + R_i$, where $k_i$ and $R_i$ are the proportional factor and base value for the resistances, respectively, and $f_i$ represents the current through resistance $r_i$. We assume the parameter value for the springs, i.e., the spring constants are unknown, and modeled using the uncertainty representation we have discussed above (Kam and Dauphin-Tanguy, 2005; Djeziri et al., 2007).

Figures 9 and 10 show the behavior of two sensor signals in the non linear and complex system. The data have been generated using a Simulink model of the system, the input is a constant force applied to the fifth mass (as shown in Figure 7).

We ran two sets of experiments to demonstrate: 1) Ability to track system behavior in the face of parameter uncertainty and 2) Fault detection studies to show that in spite of parameter uncertainty the approach would not generate a high percentage of false positives. We discuss these experiments and the results in greater detail next.

Two models have been built to do those experiments: 1) A Simulink model with uncertainty to generate data for the DBN and 2) The DBN for the nominal model to track the system behavior and perform the fault detection. The second model, the DBN for the nominal model, can be run using the nominal value for the uncertain parameters or can be run sampling those values as it has been explained in section 3.

The data used to run the experiments have different levels of uncertainty: 2% and 5% uncertainty in C parameters. There is always 2% noise attributed in every sensor.

4.2 Experimental set up

The objective of the first group of experiments performed with this approach was to check how well our designed observer tracks the behavior of a strongly non linear complex system with noise in the observations and uncertainty in the parameters. To do that, we have run the proposed particle filter based observer with
non-faulty data with sensor noise and uncertainty. The data had uncertainty (2% and 5%) in the spring constant parameter value and sensor noise of 2% in every sensor.

Following the approach presented in section 3, we used an uncertainty interval of 10% ($\delta = 0.1$) and the number of models run in each experiment is 50 ($N = 50$). Hence, 50 models with the same structure, that may differ on the actual value of their parameters, have been built. Each model has been run using a particle filter approach with 500 particles ($M = 500$). To evaluate the performance of this approach, we have compared it against the performance of the DBN of the system with the nominal parameter values. To compensate for the bias that averaging the estimation of 50 DBNs may introduce in the comparison, we have also run 50 nominal DBNs and we have generated the final estimation working out the average of the estimation over the 50 nominal DBNs.

### 4.3 Some Results

Figure 11 presents the comparison for the sensor F2 between the simulated data and the estimated data with the presented approach. The simulated data has 2% sensor noise. The same behavior can be observed in every sensor in the system (see Table 1). The method used to test the suitability of the approach has been the mean squared error (MSE). For every system sensor, we have computed the MSE of the estimation in respect to the nominal model of the system with 2% sensor noise. Table 1 summarizes the results; the MSE for each sensor appears in the table. For every system sensor, the MSE of the proposed approach is smaller, independently of the uncertainty level. Additionally, the variance of the estimation is also smaller when parameter values are sampled. A small dispersion of the estimation is important if we intent to perform fault detection, because it allows to use tighter thresholds for fault detection. As it is said in Table 1, the number of DBNs is the same for each type of experiment: 1) 50 experiments with 1 DBN each, for the nominal PF approach; 2) 1 experiment with 50 DBNs for the proposed approach.

## 5 CASE STUDY II: FAULT DETECTION

Fault detection is the first step in fault diagnosis. To perform fault detection in this piece of work the chosen statistical test has been a Ztest (Gelso et al., 2008).

The approach used to apply the Ztest is described in (Gelso et al., 2008): First of all, we define two sliding windows with sizes $N1 = 50$ and $N2=5$. The signal is assumed to have a Gaussian distribution. We use the bigger window to estimate the mean value of the assumed Gaussian distribution. The variance of the
signal distributions are estimated online using the data from the nominal simulation of the system. The residuals of the signals are used in the test, \( \mu \) parameters are their expected values (mean values) while \( \sigma \) parameters are their standard deviations.

Once we had tested that our particle filter approach is suitable to track non linear systems behavior we run some experiments to perform fault detection.

Two types of faults have been used to generate the data: 1) Abrupt faults and 2) Incipient faults.

**Abrupt fault** An abrupt fault is characterized by a fast change in a parameter value. The temporal profile of a parameter with an abrupt fault, \( p^a(t) \) is given by:

\[
p^a(t) = \begin{cases} p(t) & t < t_f \\ p(t) + b(t) = p(t) + \sigma^a_p & t \geq t_f \end{cases}
\]

where \( \sigma^a_p \) models the absolute change of the parameter value and \( t_f \) is the time when the fault happens.

**Incipient fault** An incipient fault is characterized by a slow and gradual drift term \( d(t) \) added to the nominal parameter value. The temporal profile of a parameter with an incipient fault, \( p^i(t) \) is given by:

\[
p^i(t) = \begin{cases} p(t) & t \leq t_f \\ p(t) + d(t) = p(t) + \sigma^i_p \times (t - t_f) & t > t_f \end{cases}
\]

where \( d(t) \) is the drift function modeling the variation in the parameter value and \( t_f \) is the time when the fault happens.

**5.1 Experimental set up**

As we have previously explained, two types of faults have been considered: 1) Abrupt and 2) Incipient. Abrupt faults have been considered in two different elements, a capacitance element \((K1)\) and an inductance element \((M13)\). The fault magnitude in the abrupt faults is 10\% and 25\% of the nominal value, faults of both magnitudes have been tested in both elements. Incipient faults have been considered in component \( K1 \) and its drift term is 2\%.

**5.2 Some Results**

A summary of the detection time and the results obtained from this set of experiments is presented in Table 2. Shown values are averaged on 10 repetitions. The first remarkable fact is that for the given confidence level (0.95), there are no false positives and the system is able to detect all considered faults, both abrupt and incipient. As it was expected, the detection time for incipient faults is bigger than it is for abrupt faults and also increases for incipient faults with the level of uncertainty.

Table 3 shows the number of false positives (FP) or false negatives (FN) out of 10 experiments of each type, using the nominal DBN for fault detection with uncertainty in the parameters, and for the same confidence. With the proposed approach, the number of FP and FN is zero out of 10 (the Ztest does not detect a fault when we estimate nominal behavior and it does not fail detecting a fault which has actually happened). Looking at Table 4 we can also check that above the false positives and negatives, the nominal approach also gives bigger detection time for all the tested faults, except for the incipient fault on \( K1 \) with a 5% of parameter uncertainty. However, we have to take into account that for this particular fault the nominal DBN obtains 7 false positives out of 10 trials while the sample DBN generates no false positives with the same confidence level.

These results suggest that the sample DBN could still detect smaller abrupt faults with the 0.95 confidence level, while the nominal DBN already generates an unacceptable number of false positives for this confidence level.

**6 CONCLUSIONS**

This paper has proposed a novel approach to fault detection of complex dynamic non linear systems with uncertainty on the parameters of the system model. The work assumes that the system structure is known and fixed and that the uncertainty only affects to system measurements (noise) and parameter values. The work also assumes that parameter uncertainty is additive in nature.

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**Table 1:** Mean of the MSE (and standard deviation) for each sensor in the system using 50 DBNs in each type of experiment. **Nominal** rows correspond to the results with the nominal DBN using fix values for the parameters (50 experiments with one DBN each) and **Sampled** rows correspond to the results with the proposed approach (1 experiment sampling parameters for 50 DBNs). All the figures in the table are multiply by \( 10^{-4} \).

<table>
<thead>
<tr>
<th>Sensor</th>
<th>2 % Uncertainty</th>
<th>5 % Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_2 )</td>
<td>( e_5 )</td>
</tr>
<tr>
<td><strong>Nominal</strong></td>
<td>1.81 (0.35)</td>
<td>76.2 (7.46)</td>
</tr>
<tr>
<td><strong>Sampled</strong></td>
<td>1.08 (0.05)</td>
<td>67.6 (1.29)</td>
</tr>
</tbody>
</table>

5.1 Experimental set up

5.2 Some Results

6 CONCLUSIONS
Table 2: Detection time and standard deviation (brackets) for the detection time of 10 experiments for each type of fault using the proposed approach sampling the values for the parameters with uncertainty.

<table>
<thead>
<tr>
<th>2 % Uncertainty</th>
<th>5 % Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>K1 Abr.</strong></td>
<td><strong>M3 Abr.</strong></td>
</tr>
<tr>
<td>10%</td>
<td>25%</td>
</tr>
<tr>
<td>6018.1</td>
<td>6016.0</td>
</tr>
<tr>
<td>(0.32)</td>
<td>(0)</td>
</tr>
</tbody>
</table>

Table 3: Number of false positives (FP) or false negatives (FN) for each type of fault using the nominal DBN with fixed values in the parameters.

<table>
<thead>
<tr>
<th>2 % Uncertainty</th>
<th>5 % Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nominal</strong></td>
<td><strong>K1 Abr.</strong></td>
</tr>
<tr>
<td>10%</td>
<td>25%</td>
</tr>
<tr>
<td>8 FP</td>
<td>5 FP</td>
</tr>
<tr>
<td>6018.9</td>
<td>6016.4</td>
</tr>
<tr>
<td>(0.32)</td>
<td>(0.52)</td>
</tr>
</tbody>
</table>

Additive parameter uncertainty allows a simple, elegant and systematic representation of the uncertainty on the bond graph model of the system. It also simplifies the Particle Filtering inference, because it is not necessary to propagate uncertainty of the parameters through the system equations: it is just added, exactly as the noise on the observations. The proposal is based on sampling the uncertain parameter values on an interval around its nominal values, usually well known, and averaging the estimation of several DBNs, structurally identical but with different and random parameter values.

Simulation experiments have confirmed that the approach increases the accuracy of the estate estimation respect to the estimation obtained by the nominal DBN of the system in a complex dynamic non linear system with uncertainty in the parameters and noise in the measurements.

The aforementioned improvement caters for better fault detection capabilities. This approach is perfectly able to detect the considered abrupt and incipient faults in a well-timed mode. With the same confidence level, the standard application of the nominal DBN for system tracking produce an unacceptable number of false positive and false negative detections. Moreover, detection time of incipient faults with the nominal DBN is longer than with the proposed solution.

More reasearch is going to be done related to this work, several previous works are been considered. At first time, we considered to build a DBN including the uncertain parameters as nodes. The number of parameters with uncertainties can be very big and the number of measurements in the DBN remains the same. This can be a drawback because the new DBN cannot be structurally observable (Moya et al., 2010) so it will not be able to track the system behavior properly. (Dearden and Ng, 2006) for example presents a PF application for hybrid systems with uncertainty in some parameters. The main idea in (Dearden and Ng, 2006) seems to be the estimation of parameters whose value is changing during the operation of the system. In our case, the parameters have a fixed but unknown value.

Besides the comparison with other approaches, there is still space for further research, especially considering non Gaussian distribution in parameter uncertainty, which can be easily accommodated in the Particle Filtering approach. Another promising line of research is using the Unscented Kalman Filtering to obtain a better guess of the initial state of the system.

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