# Local (Heuristic) Search Methods

## IN4082

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# Outline

"The course covers local search and its variants in combinatorial optimisation."

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- general principle
- example technique
- more techniques

# Local Search in SAT solving

**for** $i = 1$ to MAX-TRIES **do**
  $\varphi \leftarrow$ random initial assignment
  **for** $j = 1$ to MAX-STEPS **do**
    **if** $\varphi$ satisfies $\mathcal{F}$ **then**
      **return** satisfiable
    **end if**
    $\varphi \leftarrow$ FLIP$(\varphi)$
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Global, Local flips; WalkSAT, UnitWalk flips

# Local Search: General Approach

Outline:

- solutions have ‘quality’ (cost $c$)
- define *neighborhood relation* on solutions
  e.g. flipping bits in bitstrings (graphs)
- start at (random) initial solution
- explore the space of possible solutions sequentially

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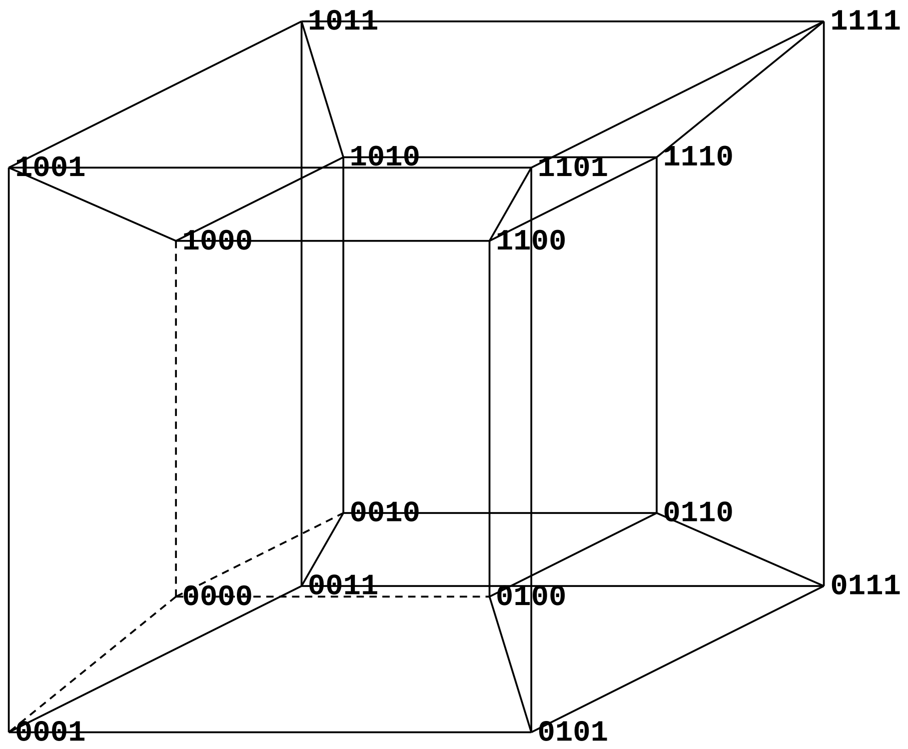
- in each step:
  - from current $S$, evaluate neighbors $S'$
  - move to $S'$ depending on $\Delta_c$ ($p$ in WalkSAT)

# Example: flipping bits





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# Landscape Metaphor

Visualization:

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Natural approach: gradient descent

- easily gets stuck in local optima
- $\rightarrow$ accept inferior neighbor with small probability

# Metropolis algorithm

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Fails to converge when close to OPT: decrease temperature over time (Simulated Annealing)

# More Local Search heuristics

Physics

- simulated annealing
- particle swarm optimization

Biology

- artificial neural networks
- artificial immune systems
- ant colony optimization

Evolution (multiple parallel solutions)

- evolutionary algorithms (GA's, GP)

# Other Topics

- easy to construct LS algorithm
- hard to get analytical performance guarantees
- evaluate performance experimentally

# LS: Best Response Dynamics

Congestion game:

- Many agents, conflicting objectives
- Cost of an edge depends on # of agents using it
- Conflict between individual vs. global
- Best Response: make a choice given others' choices
- Iteratively: Best Response Dynamics
- Stability: all strategies are best responses (Nash)
- Optimality: what is the global cost?

How does global relate to individual performance?

# LS: Combinatorial Auctions

Setup:

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LS: start with $A = (\{1, 2\}, \{13, 6\}, \ldots, \{4, 7, 10\})$