Computational Logic and Satisfiability

IN4077

Look-ahead SAT solvers

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SAT solving: DPLL

Davis Putnam Logemann Loveland [DP60,DLL62]

- Simplify (Unit Propagation)
- Split the formula
  - Variable Selection Heuristics
  - Direction heuristics

DPLL: Example

$$\mathcal{F}_{\mathrm{DPLL}} := (x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee x_3)$$
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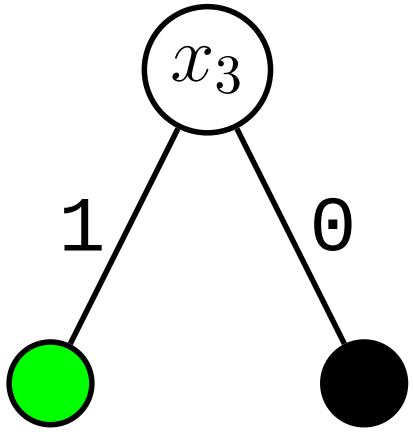

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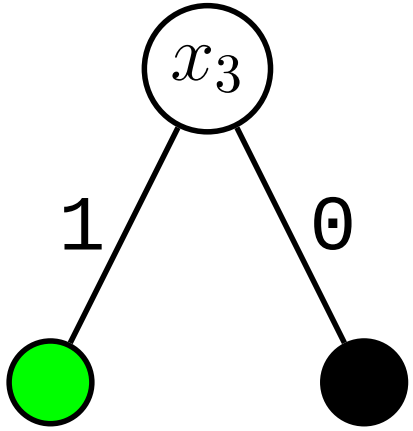

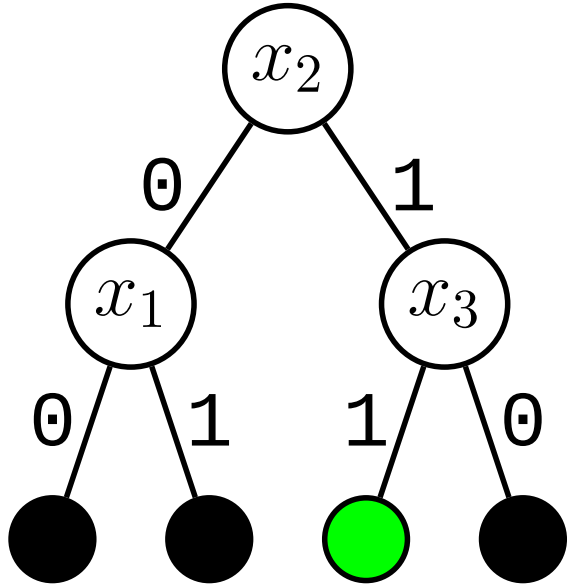

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DPLL with selection of (effective) decision variables by *look-aheads* on variables

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- Assign a variable to a truth value
- Simplify the formula

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DPLL with selection of (effective) decision variables by *look-aheads* on variables

Look-ahead:

- Assign a variable to a truth value
- Simplify the formula
- Measure the reduction
- Learn if possible
- Backtrack

Look-ahead: Example

$$\mathcal{F}_{\text{learning}} := (\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_3)$$
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Look-ahead: Properties

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- Examples: march, OKsolver, kcnfs

Look-ahead: Reduction heuristics

- Number of satisfied clauses

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- Number of implied variables

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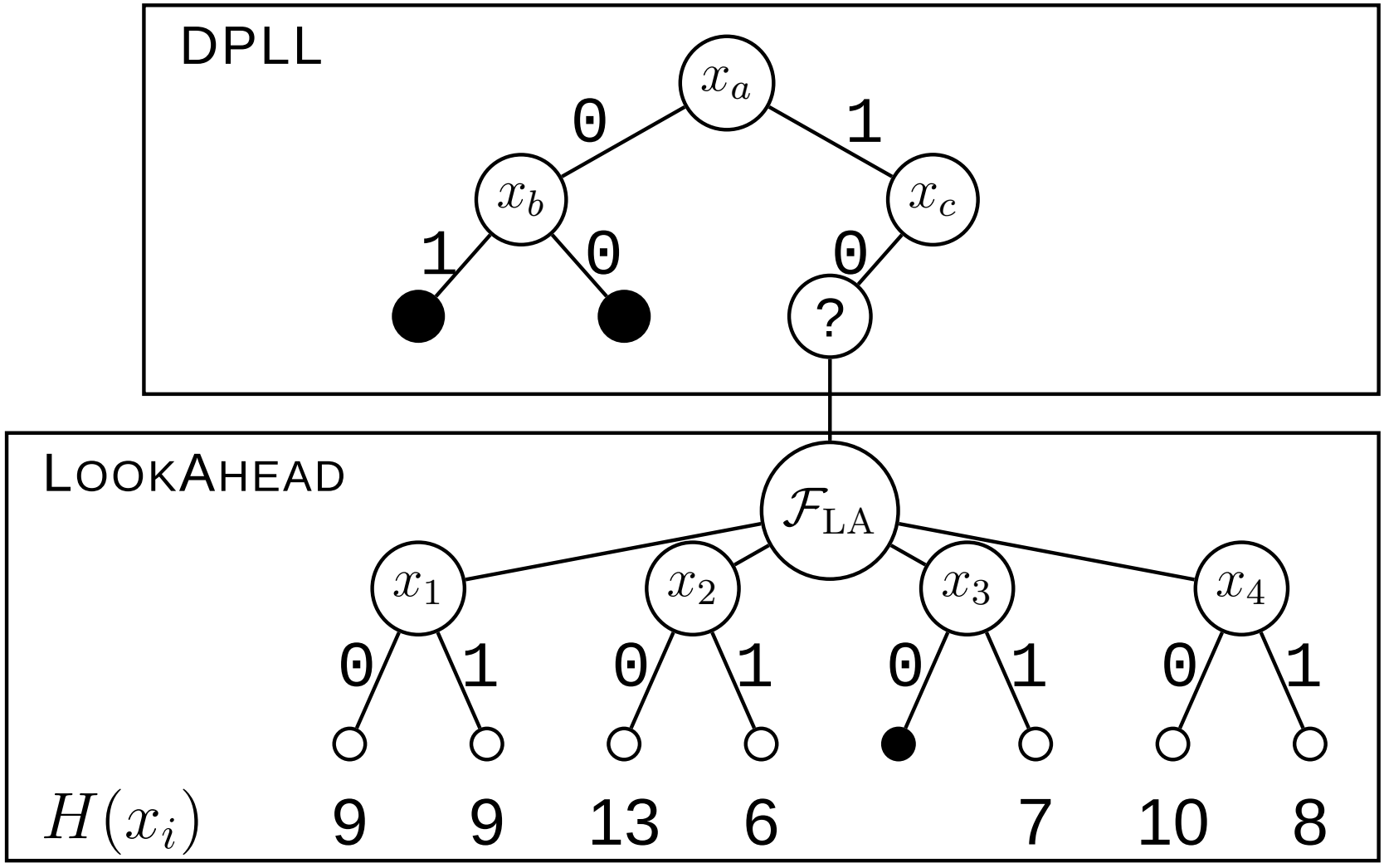
- Number of satisfied clauses
- Number of implied variables
- New (reduced, not satisfied) clauses
  - Smaller clauses more important
  - Weights based on occurring

Look-ahead: Architecture



Look-ahead: Pseudo code

1: $\mathcal{F}$:= SIMPLIFY ($\mathcal{F}$)
2: **If** $\mathcal{F}$ is empty **then return** `satisfiable`
3: **If** $\emptyset \in \mathcal{F}$ **then return** `unsatisfiable`
4: $\langle \mathcal{F};\ l_{\text{decision}} \rangle$:= LOOKAHEAD($\mathcal{F}$)
5: **if** DPLL($\mathcal{F}(l_{\text{decision}} \leftarrow 1)$) = `satisfiable` **then**
6: **return** `satisfiable`
7: **end if**
8: **return** DPLL($\mathcal{F}(l_{\text{decision}} \leftarrow 0)$)

Look-ahead: Local learning

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(local) constraint resolvents:
$(x_2 \vee x_3)$ and $(x_2 \vee \neg x_6)$

Look-ahead: Necessary assignments

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An *1-autarky* is a partial assignment that satisfies all touched clauses except one

Look-ahead: Autarky detection

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Could reduce computational cost on UNSAT

Look-ahead: 1-Autarky learning

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$$(x_1 \vee \neg x_6) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_5 \vee \neg x_6)$$

Look-ahead: 1-Autarky learning

$$\mathcal{F}_{\text{learning}} := (\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_3)$$
$$(\neg x_1 \vee x_2) \wedge (x_1 \vee x_3 \vee x_6) \wedge (\neg x_1 \vee x_4 \vee \neg x_5)$$
$$(x_1 \vee \neg x_6) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_5 \vee \neg x_6)$$

$$\varphi = \{x_2{=}0\}$$

Look-ahead: 1-Autarky learning

$$\mathcal{F}_{\text{learning}} := (\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_3)$$
$$(\neg x_1 \vee x_2) \wedge (x_1 \vee x_3 \vee x_6) \wedge (\neg x_1 \vee x_4 \vee \neg x_5)$$
$$(x_1 \vee \neg x_6) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_5 \vee \neg x_6)$$

$$\varphi = \{x_2{=}0, x_1{=}0\}$$

Look-ahead: 1-Autarky learning

$$\mathcal{F}_{\text{learning}} := (\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_3)$$
$$(\neg x_1 \vee x_2) \wedge (x_1 \vee x_3 \vee x_6) \wedge (\neg x_1 \vee x_4 \vee \neg x_5)$$
$$(x_1 \vee \neg x_6) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_5 \vee \neg x_6)$$

$$\varphi = \{x_2{=}0, x_1{=}0, x_6{=}0\}$$

Look-ahead: 1-Autarky learning

$$\mathcal{F}_{\text{learning}} := (\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_3)$$
$$(\neg x_1 \vee x_2) \wedge (x_1 \vee x_3 \vee x_6) \wedge (\neg x_1 \vee x_4 \vee \neg x_5)$$
$$(x_1 \vee \neg x_6) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_5 \vee \neg x_6)$$

$$\varphi = \{x_2{=}0, x_1{=}0, x_6{=}0, x_3{=}1\}$$

Look-ahead: 1-Autarky learning

$$\mathcal{F}_{\text{learning}} := (\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_3)$$
$$(\neg x_1 \vee x_2) \wedge (x_1 \vee x_3 \vee x_6) \wedge (\neg x_1 \vee x_4 \vee \neg x_5)$$
$$(x_1 \vee \neg x_6) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_5 \vee \neg x_6)$$

$$\varphi = \{x_2{=}0, x_1{=}0, x_6{=}0, x_3{=}1\}$$

(local) 1-autarky resolvents:
$(\neg x_2 \vee \neg x_4)$ and $(\neg x_2 \vee \neg x_5)$

Computational Logic and Satisfiability

IN4077

Look-ahead SAT solvers

Marijn J.H. Heule

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